

AUTOMORPHISM GROUPS OF CERTAIN DOMAINS IN \mathbb{C}^n WITH A SINGULAR BOUNDARY

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In this paper, we show how to use the so-called scaling technique to prove the compactness of the automorphism groups of bounded strictly convex circular domains in \mathbb{C}^n whose boundaries are not entirely smooth, in case the singular locus of the boundary is globally complicated but locally simple in some topological sense.

1. Introduction. We develop a certain scheme of computing the automorphism groups of the bounded circular convex domains in \mathbb{C}^n whose boundary is not entirely smooth. As an application, we compute the automorphism group of the unit open ball with respect to the minimal complex norm in \mathbb{C}^n introduced by K. T. Hahn and P. Pflug [3], thus answering their question raised there. In this paper, we restrict ourselves to the automorphism groups of Hahn-Pflug examples. However, we believe that all the ideas and complexity of our technique are clearly shown in this somewhat special case.

Hahn and Pflug ([3]) showed that the complex norm N^* in \mathbb{C}^n defined by

$$N^*(z_1, \dots, z_n) = \frac{1}{\sqrt{2}} \sqrt{\sum_{j=1}^n |z_j|^2 + \left| \sum_{j=1}^n z_j^2 \right|}$$

is the smallest complex norm in \mathbb{C}^n , that extends the real Euclidean norm in the following sense: *For any complex norm N in \mathbb{C}^n that extends the real Euclidean norm and satisfies the inequality $N(z) \leq |z|$ for any $z \in \mathbb{C}^n$, $N^*(z) \leq N(z)$ holds for all $z \in \mathbb{C}^n$.*

Denote by

$$B_n^* := \{z \in \mathbb{C}^n \mid N^*(z) < 1\}$$

the unit open ball in \mathbb{C}^n with respect to the norm N^* . Let $O(n, \mathbb{R})$ denote the set of all $n \times n$ real orthogonal matrices. Notice that the boundary ∂B_n^* is not entirely smooth. It was shown in [3] that this domain is not homogeneous, but no explicit description beyond that was known except when $n = 2$. Moreover, the method used in [3] to