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ROTATIONAL ENTROPY FOR ANNULUS ENDOMORPHISMS

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We study how some dynamical properties on a homeomorphism of the annulus affects its rotation set. We also introduce a topological invariant, designated "rotational entropy," that basically measures the rotational complexity of an annulus mapping.

0. Introduction. Poincaré associated to each orientation preserving homeomorphism of the circle a number, designated *rotation number*, that quanitifies the asymptotic behavior of different orbits. A natural extension of Poincaré rotation number for annuli and tori has been considered by Franks and Hermann among others.

For homeomorphisms of the annulus different orbits may rotate at different speeds. Furthermore there are orbits whose rotational behavior is so chaotic that one cannot associate a single number to its "wrapping." In general, the rotation of each orbit is captured by its *rotation interval*. The union of all rotation intervals of a map is designated the *rotation set* of the given mapping.

Franks in [**12**] proved that any orbit of an annulus homeomorphism, isotopic to the identity, with finitely many periods has rotation number. Therefore examples with chaotic rotations are expected to be dynamically complicated. Topological entropy is a topological invariant that roughly tells how many different orbits a map has. However we may have a positive entropy homeomorphism with trivial rotation set. Bowen's definition of topological entropy suggests a natural way of measuring the chaotic rotation of a given homeomorphism. We designate such a number "rotational entropy."

In this paper we start by reviewing a construction of an orientation preserving annulus homeomorphism (cf. [**4**, **15**]) with nontrivial rotation intervals. We imbed in the annulus a horseshoe in such a way that many orbits in the invariant Cantor set have "unpredictable" rotation; the rotation of the whole Cantor set is nevertheless obtained by the rotation of just one orbit. In the proof of this last assertion the *shadowing property* plays an important role.

In §3 we define "rotational entropy" and prove that it is a topological invariant closely related to the rotation set. More precisely, we prove