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THE SPACE OF INFINITE-DIMENSIONAL COMPACTA AND OTHER TOPOLOGICAL COPIES OF $(l_f^2)^\omega$

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To Doug Curtis, on the occasion of his retirement

We show that there exists a homeomorphism from the hyperspace of the Hilbert cube Q onto the countable product of Hilbert cubes such that the $\geq k$-dimensional sets are mapped onto $B^k \times Q \times Q \times \cdots$, where B is the pseudoboundary of Q. In particular, the infinite-dimensional compacta are mapped onto B^ω, which is homeomorphic to the countably infinite product of l_f^2. In addition, we prove for $k \in \{1, 2, \ldots, \infty\}$ that the space of uniformly $\geq k$-dimensional sets in 2^Q is also homeomorphic to $(l_f^2)^\omega$.

1. Introduction. If X is a compact metric space then 2^X denotes the hyperspace of X equipped with the Hausdorff metric. According to Curtis and Schori [**6**] 2^X is homeomorphic to the Hilbert cube Q whenever X is a nontrivial Peano continuum.

Our primary interest is the subset of 2^Q consisting of all infinite-dimensional compacta. This space is an $F_{\sigma\delta}$-set in 2^Q and one may expect that it is homeomorphic to the countable product of the pre-Hilbert space

$$l_f^2 = \{x \in l^2 : x_i = 0 \text{ for all but finitely many } i\}.$$

We prove this conjecture. The space $(l_f^2)^\omega$ is in a sense maximal in the class $\mathscr{F}_{\sigma\delta}$ of absolute $F_{\sigma\delta}$-spaces and it has received a lot of attention in recent years because of its topological equivalence to numerous function spaces, see e.g. Dijkstra et al. [**7**].

For $k \in \{0, 1, 2, \ldots, \infty\}$ we let $\mathrm{Dim}_{\geq k}(X)$ denote the subspace consisting of all $\geq k$-dimensional elements of 2^X. We define $\mathrm{Dim}_k(X)$ and $\mathrm{Dim}_{\leq k}(X)$ in the same way. Let $\overline{\mathrm{Dim}}_{\geq k}(X)$ stand for all uniformly $\geq k$-dimensional compacta in 2^X, i.e. spaces such that every nonempty open subset is at least k-dimensional. The default value here is $X = Q$, i.e., $\mathrm{Dim}_{\geq k} = \mathrm{Dim}_{\geq k}(Q)$ etc.

Let I stand for the interval $[0, 1]$. The Hilbert cube is denoted by $Q = \prod_{i=1}^\infty I$ with metric $d(x, y) = \max\{2^{-i}|x_i - y_i| : i \in \mathbf{N}\}$. The pseudointerior of Q is $s = \prod_{i=1}^\infty (0, 1)$ and $B = Q \setminus s$ is the pseudoboundary.