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ON THE REPRESENTATION OF THE DETERMINANT OF HARISH-CHANDRA'S C-FUNCTION OF $\mathrm{SL}(n, \mathbb{R})$

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This paper gives the explicit representation of the determinant of the Harish-Chandra C-function of $\mathrm{SL}(n, \mathbb{R})$ $(n \geq 3)$ and some application.

1. Introduction. Let G be a semisimple Lie group with finite center, K a maximal compact subgroup of G. Let θ be the Cartan involution of G fixing K. Let P be a cuspidal parabolic subgroup and $P = MAN$ its Langlands decomposition. For σ in $\widehat{M_d}$ and γ in $\widehat{K}$, we set $\tau = (\gamma, \gamma)$ and denote the space of the τ_M-spherical cusp forms on M by ${}^0\mathfrak{C}_M(M, \tau_M)$. The Harish-Chandra C-function $C_{\overline{P}|P}(1 : \nu)$ has important information in the representation theory.

In the determinant of $C_{\overline{P}|P}(1 : \nu)$, L. Cohn has proved the following results.

THEOREM (*see* [2], *p.* 129). *There exist functions* $\mu_1, \dots, \mu_r \in \mathfrak{a}^*$ *and constants* $p_{i,j}$, $q_{i,j}$ $(i = 1, \dots, r,\ j = 1, \dots, j_i)$ *depending on* τ *such that*

$$\det C_{\overline{P}|P}(1 : \nu) = \mathrm{const} \cdot \prod_{i=1}^{r} \prod_{j=1}^{j_i} \frac{\Gamma(\frac{\langle \nu, \alpha_i \rangle}{2\langle \mu_i, \alpha_i \rangle} + q_{i,j})}{\Gamma(\frac{\langle \nu, \alpha_i \rangle}{2\langle \mu_i, \alpha_i \rangle} + p_{i,j})},$$

where $\alpha_1, \dots, \alpha_r$ *are reduced* $\mathfrak{a}$*-roots.*

He gives a conjecture that the constants $p_{i,j}$ and $q_{i,j}$ are rational numbers and depending linearly on the highest weight of the irreducible components of τ.

Let G be $\mathrm{SL}(n, \mathbb{R})$ and P the minimal parameter subgroup of G. In the case that $n = 2$, the Harish-Chandra C-function and determinant of it are well known explicitly. If n is 3 or 4, in [4] Eguchi and the author give the explicit formula of the determinant of Harish-Chandra's C-function of G, which solves Cohn's conjecture affirmatively. The purpose of this paper is to extend the result in [4]