

ON CERTAIN IWAHORI INVARIANTS IN THE UNRAMIFIED PRINCIPAL SERIES

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An affine Hecke algebra is additively the tensor product of a finite dimensional Hecke algebra with the coordinate ring Θ of a complex torus. In this paper we give explicit formulas for eigenvectors of Θ in unramified principal series representations of the reductive p -adic group G associated to \mathcal{H} . This leads to new information about intertwining operators, Jacquet modules and submodules of principal series representations.

Let G be a reductive p -adic group, τ an unramified character of a minimal parabolic subgroup P , and $I(\tau) = \text{ind}_P^G \tau$ the induced principal series representation of G . The space $I(\tau)^B$ of vectors in $I(\tau)$ which are invariant under an Iwahori subgroup B affords a representation of the affine Hecke algebra \mathcal{H} corresponding to G . It is known that taking B -invariants yields an equivalence of categories between (admissible G -modules generated by their B -invariants) and (finite dimensional \mathcal{H} modules). Thus the representation theory of $I(\tau)$ is captured by the action of \mathcal{H} on $I(\tau)^B$. The irreducible representations of \mathcal{H} have been classified ([K-L] and [G]). However, the decomposition of $I(\tau)^B$ itself, though well studied (see the references), is not completely understood. The purpose of this paper is to describe certain functions in $I(\tau)^B$ which are important for finding the submodules of this representation explicitly. This investigation enables us to extend some results of Rodier [R] and Rogawski [Ro], and also yields a new proof of the irreducibility criterion for $I(\tau)$ due to Kato and Müller ([Ka], [M]).

Recall (cf. [L]) that as a vector space, \mathcal{H} is the tensor product of two subalgebras

$$\mathcal{H} = \Theta \otimes \mathcal{H}_W,$$

where \mathcal{H}_W is the Hecke algebra of the finite Weyl group W of G and Θ is isomorphic to the coordinate ring of a maximal torus T in the complex Lie group which is dual to G in the sense of Langlands. As a \mathcal{H}_W -module, $I(\tau)^B$ is always the regular representation of \mathcal{H}_W , so as τ varies, the change in the structure of $I(\tau)^B$ is seen in the action of