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REDUCTION OF TOPOLOGICAL STABLE RANK IN INDUCTIVE LIMITS OF C^*-ALGEBRAS

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We consider inductive limits A of sequences $A_1 \to A_2 \to \cdots$ of finite direct sums of C^*-algebras of continuous functions from compact Hausdorff spaces into full matrix algebras. We prove that A has topological stable rank (tsr) one provided that A is simple and the sequence of the dimensions of the spectra of A_i is bounded. For unital A, $\mathrm{tsr}(A) = 1$ means that the set of invertible elements is dense in A. If A is infinite dimensional, then the simplicity of A implies that the sizes of the involved matrices tend to infinity, so by general arguments one gets $\mathrm{tsr}(A_i) \leq 2$ for large enough i whence $\mathrm{tsr}(A) \leq 2$. The reduction of tsr from two to one requires arguments which are strongly related to this special class of C^*-algebras.

The problem of reduction of real rank (see [6]) for these algebras was recently studied in [2] in connection with some interesting features revealed in several papers ([3], [1], [15], [5], [12], [11]). The reduction of tsr and real rank for other classes of C^*-algebras was studied in [22], [21], [8], [24], [17], [25].

The paper consists of three sections:

1. Preliminaries and Notation
2. Local aspects of the connecting homomorphisms
3. The Main Result.

1.

1.1. For a unital C^*-algebra A and a finitely generated projective A-module E, we denote by $\mathrm{End}_A(E)$ the algebra of A-linear endomorphisms of E and by $\mathrm{GL}_A(E)$ the group of units of $\mathrm{End}_A(E)$. For $E = A^n$ we shall write $\mathrm{GL}(n, A)$ for $\mathrm{GL}_A(A^n)$ and $\mathrm{GL}^0(n, A)$ for the connected component of 1. Let $\mathrm{U}(A)$ denote the unitary group of A and $\mathrm{U}(n) := \mathrm{U}(\mathbf{C}^n)$. A selfadjoint idempotent element of a C^*-algebra will be simply called projection.

Recall some definitions from [23]. For a unital C^*-algebra A and a natural number n let $Lg_n(A)$ denote the set of n-tuples of elements of A which generate A as a left ideal. The topological stable rank of A is the least n (if it does not exist it will be taken by definition