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COMPACT OPERATIONS, MULTIPLIERS AND RADON-NIKODYM PROPERTY IN JB^*-TRIPLES

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We study the (weak) compactness of certain algebraic operations on JB^*-triples and we introduce multiplier triples. Applications to structure theory are given and connections with the Radon-Nikodym Property are described.

Introduction. Recently the authors [5] studied the Radon-Nikodym property (RNP) in the dual spaces of some complex Banach spaces known as JB^*-triples. A number of intrinsic characterisations were obtained. One of these was that, if A is a JB^*-triple, then A^* has the RNP if and only if A has a composition series of closed triple ideals (i.e. M-ideals) for which successive quotients can be realised either as spaces of compact operators from one Hilbert space to another or else are reflexive. This hints at a connection between the RNP and compact, and weakly compact, operators on A itself. This paper evolves from an investigation into the form and extent of this connection.

Thus, in a fairly systematic way, we study the (weak) compactness of natural algebraic operations, introduce the notion of a multiplier triple of a JB^*-triple (which may be of independent interest), and explain how the resulting phenomena interweave with the RNP.

JB^*-triples originate in the study of holomorphy in unspecified (possibly infinite) dimension and can be realised as that class of complex Banach spaces whose unit ball is a bounded symmetric domain (in finite dimensions, the classical Cartan domains of complex analysis) [23]. The considerable recent activity and rapid progress in JB^*-triples is due in no small part to fertile applications in, amongst other topics (see [26, 27]), infinite dimensional Lie algebras, mathematical physics and operator spaces. Notably, the image of a contractive projection on a C^*-algebra is, while rarely a C^*-algebra, always a JB^*-triple [15].

1. Preliminaries. Precisely a *JB^*-triple* is a complex Banach space A with a continuous triple product $\{\dots\}: A^3 \to A$ which is linear and symmetric in the outer variables and antilinear in the middle variable, and satisfies