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LUSTERNIK-SCHNIRELMANN INVARIANTS IN PROPER HOMOTOPY THEORY

R. AYALA, E. DOMÍNGUEZ, A. MÁRQUEZ, AND A. QUINTERO

We introduce and study proper homotopy invariants of the Lusternik-Schnirelmann type, p-cat$(-)$, p-Cat$(-)$, and cat$\varepsilon(-)$ in the category of T_2-locally compact spaces and proper maps. As an application, $\mathbb{R}^n$ $(n \neq 3)$ is characterized as (i) the unique open manifold X with p-Cat$(X) = 2$, or (ii) the unique open manifold with one strong end and p-cat$(x) = 2$.

Introduction. The category cat(X) of a space X in the sense of Lusternik and Schnirelmann (L-S category) is the smallest number k such that there exists an open covering $\{X_1, \dots, X_k\}$ of X for which each inclusion $X_j \subseteq X$ is nullhomotopic in X. This concept was introduced by the quoted authors in their studies on calculus of variations **[16]** and they used it as a lower bound for the number of critical points of a differentiable real function on a manifold. The basic work on the homotopical significance of cat is due to Borsuk (see **[5]**). Borsuk's work was continued by Fox **[10]**.

Here we present the definition and the basic properties of a new numerical topological invariant for T_2-locally compact spaces which agrees with the notion of L-S category for T_2-compact spaces. This invariant, denoted p-cat(X), is called the proper L-S category of X and turns out to be a proper homotopy invariant of X. Hence, p-cat(X) is a finer invariant than cat(X).

In **[10]** several generalizations of L-S category are suggested. More explicitly, a general notion of L-S $\mathscr{A}$-category with respect to a class $\mathscr{A}$ of spaces has been developed by Puppe and Clapp in **[6]**. Our work shares some common points with **[6]** but does not fit into the notion of L-S $\mathscr{A}$-category since we entirely deal with proper maps instead of ordinary continuous maps.

Another generalization of L-S category has been given in **[1]**, where L-S category for pro-objects in pro-$\mathscr{T}\!op$ is defined. This idea is related to proper L-S category by the Edwards-Hastings embedding (see **[8]**) which provides a close link between proper homotopy theory and homotopy in pro-$\mathscr{T}\!op$.