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AFFINE LAMINATION SPACES FOR SURFACES

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In this paper we extend Thurston's space $\mathscr{PL}(M)$ of projective classes of measured laminations in the compact surface M to a space $\mathscr{AL}(M)$ of laminations with transverse affine structures. The main theorem is that $\mathscr{AL}(M)$ is homeomorphic to the product $\mathscr{PL}(M) \times H^1(M; \mathbb{R})$.

To avoid confusion, it should be pointed out at the outset that the term "affine lamination" can have two different meanings, depending on whether the transverse affine structure is defined in the ambient surface or just in a neighborhood of the lamination. It is the former, stronger, notion that we are interested in here. Such ambient affine structures can be regarded as transverse affine structures on singular foliations of M.

The topology on $\mathscr{AL}(M)$ is defined via length functions on homologically trivial loops in M, by the following procedure. The obstruction to an affine lamination $L \in \mathscr{AL}(M)$ being a measured lamination is a holonomy homomorphism $\sigma_L: \pi_1(M) \to \mathbb{R}_+$, which measures the amount by which arcs transverse to L are stretched or shrunk as they are transported around loops in M. Since $\mathbb{R}_+$ is abelian, commutators in $\pi_1(M)$ have trivial holonomy, so the lift $\widehat{L}$ of L to the universal abelian cover $\widehat{M}$ of M, corresponding to the commutator subgroup of $\pi_1(M)$, has a transverse measure, unique up to scalar multiplication. Let $\widehat{\mathscr{AL}}(M)$ denote the unprojectivized version of $\mathscr{AL}(M)$, consisting of the measured laminations $\widehat{L} \subset \widehat{M}$ constructed in this way. Loops γ in $\widehat{M}$ determine length functions $l_\gamma: \widehat{\mathscr{AL}}(M) \to [0, \infty)$, with $l_\gamma(L)$ as usual the infimum of the length, with respect to the transverse measure on $\widehat{L}$, of loops in $\widehat{M}$ homotopic to γ. These l_γ's are the coordinates of a function $l: \widehat{\mathscr{AL}}(M) \to [0, \infty)^{\mathscr{C}}$, $\mathscr{C}$ being the set of free homotopy classes of loops in $\widehat{M}$. We prove l is injective, and give $\widehat{\mathscr{AL}}(M)$ the induced topology. Projectivizing this yields $\mathscr{AL}(M)$.

We then determine the global topology of $\mathscr{AL}(M)$. The most