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A MEASURE THEORETICAL PROOF OF THE CONNES-WOODS THEOREM ON AT-FLOWS

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It was shown by A. Connes and J. Woods that every ITPFI factor of type III_0 is characterized to be an AFD factor whose flow of weights is conservative, aperiodic, and approximately transitive (AT). In this paper, a measure theoretical proof of their result will be shown from the side of ergodic theory, without using modular theory.

1. Introduction. ITPFI factors, introduced by Araki and Woods [**1**], provide us concrete models of von Neumann algebras. Among approximately finite dimensional (AFD) factors of type III_0, their exact position was characterized by Connes and Woods [**1**], whose result says the flow of weights associated with an ITPFI factor is conservative, aperiodic, and approximately transitive (AT), and conversely. As every ITPFI factor is the Krieger factor arising from a product odometer action with a product measure, and as the isomorphic classes of AFD factors of type III_0 correspond bijectively with the orbit equivalence classes of ergodic amenable actions of type III_0 by countable groups of non-singular transformations, their result in effect says that an ergodic amenable action of type III_0 by a countable group of non-singular transformations is orbit equivalent with a product odometer action if and only if its associated flow is conservative, aperiodic, and AT (see Definition 17). In such a measure theoretical setting, the one direction that product odometer action implies AT was proved directly by Hawkins [**6**].

In this paper we would like to present a purely measure theoretical proof of the other direction, which seems to be more difficult. The proof is based on the following observations. Given a countable group G of type III_0, ergodic, non-singular transformations, a transformation group $\mathscr{G}$ is introduced (§3). $\mathscr{G}$ is orbit equivalent with G when the action of G is amenable. $\mathscr{G}$ is equipped with all the information available from the AT-property of the associated flow of G, and it is easier to check that $\mathscr{G}$ is a product odometer action rather than to check G. Our approach might be helpful for the reader not familar with modular theory of von Neumann algebras, and the notion of