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COBCAT AND SINGULAR BORDISM

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Dold proved that a homomorphism $\phi: H^n(\mathrm{BO}) \to \mathbb{Z}_2$ corresponds to a manifold M^n if and only if $\phi(\mathrm{Sq}^p u + v_p \cdot u) = 0$, $\forall p \geq 0$ and $\forall u \in H^{n-p}(\mathrm{BO})$, v_p being the Wu class. The object of the present work is to have a singular analogue of this result and to study the bordism classification of singular manifolds in BO.

1. Introduction. Singh [1] has developed the notion of cobcat for a manifold M^n and has classified, upto bordism, all manifolds M^n with $\mathrm{cobcat}(M^n) \leq 3$. $\mathrm{Cobcat}(M^n)$ was defined to be the smallest positive integer k such that the number $\langle W_{i_1} \cdots W_{i_p}, [M^n] \rangle = 0$ for all partitions $i_1 + \cdots + i_p = n$ with $k \leq p \leq n$.

Here we develop the notion of cobcat for a singular manifold (M^n, f) in a space X and discuss the bordism classification of all singular manifolds (M^n, f) in BO with $\mathrm{cobcat}(M^n, f) \leq 3$, $n = 2^r$.

Here all the manifolds are to be unoriented, smooth and closed, and all the homology and cohomology coefficients are to be in $\mathbb{Z}_2$. The space X is such that for each n, $H_n(X)$ and hence $H^n(X)$ is a finite dimensional vector space over $\mathbb{Z}_2$.

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2. Preliminaries. Consider the set $N_n(X)$ of bordism classes of n-dimensional singular manifolds (M^n, f) in X, $f: M^n \to X$ being a continuous map. We know that $N_n(X)$ is an abelian group under the operation "disjoint union"

$$[M_1^n, f_1] + [M_2^n, f_2] = [M_1^n \sqcup M_2^n, f_1 \sqcup f_2],$$

where $f_1 \sqcup f_2: M_1^n \sqcup M_2^n \to X$ is given by

$$f_1 \sqcup f_2(x) = \begin{cases} f_1(x) & \text{if } x \in M_1^n, \\ f_2(x) & \text{if } x \in M_2^n. \end{cases}$$

Further, we have

$$N_*(X) = \bigoplus_{n \geq 0} N_n(X).$$