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ON SIX-CONNECTED FINITE H-SPACES

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In this note we shall prove the following theorem.

MAIN THEOREM. ***Let X be a 6-connected finite H-space with associative* mod 2 *homology. Further, suppose that* $\mathrm{Sq}^4 H^7(X; Z_2) = 0$ *and* $\mathrm{Sq}^{15} H^{15}(X; Z_2) = 0$. *Then X is either contractible or has the homotopy type of a product of seven-spheres.***

0. Introduction. It should be noted that there are several results related to this theorem. Lin showed that any finite H-space with associative mod 2 homology has its first nonvanishing homotopy in degrees 1, 3, 7, or 15 (or is contractible). A seven-sphere is an H-space, but not a mod 2 homotopy-associative one [**4**, **10**]. Further work of Hubbuck [**5**], Sigrist and Suter [**12**], and others has shown that spaces whose mod 2 cohomology has the form

$$\Lambda(x_7, x_{11}) \quad \text{or} \quad \Lambda(x_7, x_{11}, x_{13})$$

are not realizable as H-spaces. (Here x_i denotes an element of degree i.) One is led to conjecture that

Conjecture 1. Every two-torsion-free 6-connected finite H-space is homotopy equivalent to a product of seven-spheres (or is acyclic).

Conjecture 2. Every two-torsion-free homotopy-associative 6-connected finite H-space is acyclic.

Conjecture 1 implies Conjecture 2 by [**4**, **11**].

Henceforth, X will denote an H-space that satisfies the hypotheses of the Main Theorem, and $H^*(X)$ will denote $H^*(X; Z_2)$. The proof of the Main Theorem will be accomplished in a series of steps, which we record here. Our goal is to show that under the hypotheses, X has mod 2 cohomology an exterior algebra on 7-dimensional generators. This relies heavily on the following theorem.

Steenrod Connections [**8**]. Let X be a finite simply-connected H-space with associative mod 2 homology. Then for $r \geq 0$, $k > 0$,