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A GENERALIZATION OF MAXIMAL FUNCTIONS ON COMPACT SEMISIMPLE LIE GROUPS

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Let G be a compact semisimple Lie group with finite centre. For each positive number s, let μ_{sH} denote the $\mathrm{Ad}(G)$-invariant probability measure carried on the conjugacy class of $\exp(sH)$ in G. With this one-parameter family of measures, we define the maximal operator $\mathscr{M}_H$ on $\mathscr{C}(G)$. We then estimate the Fourier transform of μ_{sH} and of some derived distributions. Our result leads to the boundedness of $\mathscr{M}_H$ on $L^p(G)$, for all p greater than some index p_0 in $(1, 2)$. This generalizes a recent result of M. Cowling and C. Meaney [2].

Introduction. Let G be a compact semisimple Lie group of rank l with finite centre, and with its Haar measure normalized to have total mass 1. Let $\mathfrak{g}$ denote its Lie algebra, and let $\mathfrak{h}$ be a maximal toral subalgebra of $\mathfrak{g}$. We denote by Φ the root system of $(\mathfrak{g}^c, \mathfrak{h}^c)$, and fix $\Delta = \{\alpha_j : j \in I\}$, where $I = \{1, \dots, l\}$, to be a base of Φ (as in [**3**, §10.1]). With respect to Δ, we write Φ^+ for the set of positive roots, whose members are of the form

$$\alpha = \sum_{j \in I} n_j(\alpha)\alpha_j,$$

with $n_j(\alpha) \in \mathbf{Z}^+ \cup \{0\}$ for all $j \in I$, and Λ^+ for the set of dominant weights, which parametrizes the dual object of G.

We equip the Lie algebra $\mathfrak{g}$ with the positive definite inner product $(\cdot, \cdot)$ derived from the Killing form. For each $\nu \in \mathfrak{h}^*$, we define $H_\nu \in \mathfrak{h}$ by

$$\nu(H) = (H_\nu, H) \quad \forall H \in \mathfrak{h}.$$

We also transfer the inner product to $\mathfrak{h}^*$ via

$$(\nu, \nu') = (H_\nu, H_{\nu'}) \quad \forall \nu, \nu' \in \mathfrak{h}^*.$$

The norm on $\mathfrak{h}^*$ and $\mathfrak{h}$, induced by these inner products, will then be denoted by $|\cdot|$.

We choose a regular element $H \in \mathfrak{h}$, for which $\alpha(H) \neq 0$ for all $\alpha \in \Phi^+$, and fix $R > 0$ such that $\exp(sH)$ is regular in G for any