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ON SOME PROPERTIES OF EXHAUSTION MAPS BETWEEN BOUNDED DOMAINS

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In this paper, we study the properties of exhaustion maps defined on Thullen domains and piecewise smooth intersection domains. In general, "exhaustion between two bounded domains" is a much weaker condition than biholomorphism, but in the above two cases, we are able to show that the two concepts are almost the same. In particular, we generalize Pinchuk's theorems to the case of exhaustion maps in the domains mentioned above.

1. Introduction and statement of results. It is well known that two bounded domains in $\mathbb{C}^n$, $n > 1$, are rarely biholomorphically equivalent to each other. In 1907, Poincaré showed that the ball and the polydisc in $\mathbb{C}^2$ were not biholomorphic to each other. This result was strengthened by Henkin [7] and Pinchuk [**10**] to bounded domains with various boundary regularity and convexity conditions. In particular, they have the following remarkable results:

THEOREM A ([**10**]). *Let D be a convex but not strictly pseudoconvex domain in $\mathbb{C}^n$ with boundary of class $\mathscr{C}^{2+\varepsilon}$, $\varepsilon > 0$. Then D cannot be biholomorphic to any bounded strictly pseudoconvex domain.*

THEOREM B ([7, **10**]). *Let D be a bounded pseudoconvex domain in $\mathbb{C}^n$, with piecewise $\mathscr{C}^2$ smooth (see the definition below) but not smooth boundary. Then D cannot be biholomorphic to any bounded domain with $\mathscr{C}^2$ boundary.*

By a domain D with piecewise $\mathscr{C}^k$ smooth boundary, we mean that in some neighborhood U of $\overline{D}$, there exist real functions ρ_ν defined on it, where $\nu = 1, \ldots, m$, such that

(a) $\partial D \subset \bigcup_1^m S_\nu$, where $S_\nu = \{z\colon \rho_\nu(z) = 0\}$;

(b) for any subset $\{i_1, \ldots, i_q\} \subset \{1, \ldots, m\}$, $d\rho_{i_1} \wedge \cdots \wedge d\rho_{i_q} \neq 0$, at every point where $\rho_{i_1} = \cdots = \rho_{i_q} = 0$.

In this paper, we study the conditions when two domains are "almost equivalent" in the sense of exhaustion as defined below.

DEFINITION. Let D, G be bounded domains in $\mathbb{C}^n$. We say that "G can be exhausted by D", or "D exhausts G", if for every