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LOCAL REAL ANALYTIC BOUNDARY REGULARITY OF AN INTEGRAL SOLUTION OPERATOR OF THE $\overline{\partial}$-EQUATION ON CONVEX DOMAINS

ZHENHUA CHEN

In this paper we show that a well known integral solution operator of the $\overline{\partial}$-equation on a convex domain Ω locally preserves the real analyticity of $\overline{\partial}$-closed $(0, 1)$ forms at boundary points near which $\partial\Omega$ is totally convex in the complex tangential directions.

1. Introduction. The real analytic boundary regularity of the canonical solution or Kohn's solution of the $\overline{\partial}$-equation was studied by many researchers [2], [3], [7], [8], [9], [10], [14], [15], [16]. In general, the canonical solution is not explicit. An interesting question is, whether we can find some explicit or computable solution of the $\overline{\partial}$-equation that is real analytically regular up to the boundary of pseudoconvex domains. In this paper we prove the following local real analytic boundary regularity of a well known integral solution operator of the $\overline{\partial}$-equation for totally convex domains (terminology will be defined in §2). The real analytic boundary regularity of the canonical solution of such domains has not been proved yet. The recent work of Boas and Straube [1] gives the global C^∞ boundary regularity of the canonical solution for convex domains.

THEOREM. *Suppose Ω is a bounded convex domain in $\mathbb{C}^n$ with C^2 boundary. If p is a real analytic boundary point of Ω, and $\partial\Omega$ is totally convex at p in the complex tangential directions, then the Henkin operator T locally preserves the real analyticity of $\overline{\partial}$-closed $(0, 1)$ forms up to the boundary point p.*

It had been conjectured for some time that global analytic hypoellipticity of the $\overline{\partial}$-Neumann problem held for weakly pseudoconvex domains, and this remains open. But local results on finite type domains came as a surprise comparing with C^∞ results. A recent counterexample of Christ and Geller [6] shows that local real analytic boundary regularity of the $\overline{\partial}$-equation does not hold for general pseudoconvex domains of finite type. We would like to point out that the domain