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ASYMPTOTIC BEHAVIOR OF EIGENVALUES FOR A CLASS OF PSEUDODIFFERENTIAL OPERATORS ON $\mathbf{R}^n$

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We consider a pseudodifferential operator P whose symbol has an asymptotic expansion by quasi homogeneous symbols and the principal symbol is degenerate on a submanifold. Under appropriate conditions, P has the discrete spectrum. Then we can get the asymptotic behavior of the counting function of eigenvalues of P with remainder estimate according to various cases.

0. Introduction. We consider the asymptotic behavior of eigenvalues for a class of pseudodifferential operators on $\mathbf{R}^n$ containing the Schrödinger operator with magnetic field:

$$\begin{aligned}(0.1)\quad p^w(x, D) &= H(a) + V(x)\\ &= \sum_{j=1}^{n}\left(\frac{1}{i}\frac{\partial}{\partial x_j} - a_j(x)\right)^2 + V(x) \qquad (i = \sqrt{-1}).\end{aligned}$$

Throughout this paper we assume that the magnetic potential $a(x)$ satisfies:

$$a(x) = (a_1(x), a_2(x), \dots, a_n(x)) \in C^\infty(\mathbf{R}^n; \mathbf{R}^n)$$

and the scalar potential $V(x)$ satisfies $V(x) \in C^\infty(\mathbf{R}^n; \mathbf{R})$. We regard $p^w(x, D)$ as a linear operator in $L^2(\mathbf{R}^n)$ with domain $C_0^\infty(\mathbf{R}^n)$. Under appropriate conditions, we shall see that $p^w(x, D)$ is essentially self-adjoint in $L^2(\mathbf{R}^n)$ and its self-adjoint extension P is semibounded from below and has a compact resolvent in $L^2(\mathbf{R}^n)$. Therefore the spectrum $\sigma(P)$ of P is discrete, that is, $\sigma(P)$ consists only of eigenvalues of finite multiplicity. Thus we can denote the eigenvalues with repetition according to multiplicity by: $\lambda_1 \le \lambda_2 \le \cdots$, $\lim_{k\to\infty} \lambda_k = \infty$. We consult the asymptotic behavior of the counting function $N_P(\lambda)$ of eigenvalues:

$$(0.2)\qquad N_P(\lambda) = \#\{j\,;\, \lambda_j \le \lambda\}.$$

In the special case $a(x) = 0$, i.e., $p^w(x, D)$ is of the form:

$$(0.3)\qquad p^w(x, D) = -\Delta + V(x),$$