

INTEGRAL SPINOR NORMS IN DYADIC LOCAL FIELDS I

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The spinor norms of integral rotations on some quadratic forms over an arbitrary dyadic local field are determined. As an application, the results obtained in Bon Durant's paper are improved.

1. Introduction. Because of the absence of a local-global principle in the equivalence of integral quadratic forms over a global field, Eichler and Kneser developed the theory of spinor genera, which strongly depend on spinor norms of local integral rotations. For nondyadic cases the spinor norms are well understood (see [6]), but they become difficult to be determined when 2 is no longer a unit. Hsia [5], the author [10] computed the spinor norms of local integral rotations on modular quadratic forms completely. Earnest and Hsia [3] determined the spinor norms of integral rotations on arbitrary quadratic forms over a dyadic local field in which 2 is prime, and Bon Durant [1] recently considered the spinor norms of integral rotations over a local field which is a quadratic ramified extension field of \mathbb{Q}_2 . In the present article we first determine the spinor norms of all binary quadratic forms over an arbitrary dyadic local field, and then use these results to obtain the spinor norms of some quadratic forms over an arbitrary dyadic local field which Jordan components are one dimension. As an application, we consider the spinor norms of quadratic forms over the dyadic local fields which ramification index of 2 is 2 and improve the sufficient condition for the class number of an indefinite quadratic form over the ring of integers of a number field to be a divisor of the class number of the field.

Notation and terminology used here is that of [8]. In particular, F denotes a dyadic local field, \mathfrak{o} the ring of integers in F , \mathfrak{p} the maximal ideal of \mathfrak{o} , U the group of units in \mathfrak{o} , $e = \text{ord } 2$ the ramification index of 2 in F , π a fixed prime element in F , $D(\cdot, \cdot)$ the quadratic defect function, Δ a fixed unit of quadratic defect $4\mathfrak{o}$, V a regular quadratic space over F associated symmetric bilinear form $B(x, y)$, L a lattice on V , $O^+(V)$ the group of rotations on V , $O^+(L)$ the corresponding subgroup of units of L , and $\theta(\cdot, \cdot)$ the