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BERGMAN AND HARDY SPACES WITH SMALL EXPONENTS

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We show that for each $0 < p < 1$ the dual space of the Hardy and weighted Bergman space on the open unit ball is isomorphic to the Bloch space (with equivalent norms) under certain volume integral pairing.

1. Introduction. We present a new approach to an old problem, namely, the problem of describing the continuous linear functionals on the Bergman and Hardy spaces with $0 < p < 1$. We restrict our attention to the open unit ball in $\mathbf{C}^n$, even though our approach has the potential to generalize to bounded symmetric domains.

Let B_n be the open unit ball in $\mathbf{C}^n$ with boundary ∂B_n. Let $H(B_n)$ denote the space of all holomorphic functions in B_n. For $0 < p < +\infty$ and $\alpha > -1$ we let

$$L_a^p(B_n, dv_\alpha) = H(B_n) \cap L^p(B_n, dv_\alpha)$$

denoted the weighted Bergman space, where

$$dv_\alpha(z) = C_\alpha(1 - |z|^2)^\alpha \, dv(z).$$

Here dv is volume measure on B_n and C_α a normalizing constant so that dv_α has total mass 1. For $f \in L_a^p(B_n, dv_\alpha)$ we write

$$\|f\|_{\alpha,p} = \left[\int_{B_n} |f(z)|^p \, dv_\alpha(z)\right]^{1/p}.$$

A linear functional F on $L_a^p(B_n, dv_\alpha)$ is bounded if there exists a constant $C > 0$ such that $|F(f)| \le C\|f\|_{\alpha,p}$ for all f in $L_a^p(B_n, dv_\alpha)$. The dual space of $L_a^p(B_n, dv_\alpha)$, denoted $L_a^p(B_n, dv_\alpha)^*$, consists of all bounded linear functionals on $L_a^p(B_n, dv_\alpha)$. For each $0 < p < +\infty$ the space $L_a^p(B_n, dv_\alpha)^*$ is a Banach space with the norm

$$\|F\| = \sup\{|F(f)| : \|f\|_{\alpha,p} \le 1\}.$$

Note that $L_a^p(B_n, dv_\alpha)$ itself is not a Banach space when $0 < p < 1$.