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ON INFINITESIMAL BEHAVIOR OF THE KOBAYASHI DISTANCE

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The condition that the Kobayashi distance between two nearby points in a pseudo-convex domain is realized by the Poincaré distance on a single analytic disk joining the two points is studied. It is shown that the condition forces the Kobayashi indicatrix to be convex. Examples of pseudo-convex domains on which this condition fails to hold are given. The (infinitesimal) Kobayashi metric is shown to be a directional derivative of the Kobayashi distance. It is shown that, if the condition holds near any point of a pseudo-convex domain and if the Kobayashi metric is a complete Finsler metric of class C^2, then the Kobayashi distance between any two points in the domain can be realized by the Poincaré distance on a single analytic disk joining the two points.

1. Introduction. In this paper, we study the infinitesimal behavior of the Kobayashi distance on a pseudo-convex domain. In particular, we examine the condition that the distance between two nearby points in a pseudo-convex domain is realized by the Poincaré distance on a single analytic disk joining the two points.

Let D be a bounded domain in $\mathbf{C}^m$ and let δ denote the Poincaré distance on the open unit disk Δ in the complex plane. If D is convex, then *the Kobayashi distance* between two points $p, q \in D$ can be defined in terms of a single analytic disk joining p and q in D: the function $d^* : D \times D \to \mathbf{R}$ defined by

$$(1.1)\quad d^*(p, q) = \inf_f \{\delta(a, b) : f : \Delta \to D \text{ holomorphic}, \\ f(a) = p,\ f(b) = q,\ a, b \in \Delta\}$$

satisfies the triangle inequality, and d^* is the Kobayashi distance function on D **[L1]**.

When D is a pseudo-convex domain, d^* does not in general satisfy the triangle inequality (see **[L1]** for example). To define the Kobayashi distance $d : D \times D \to \mathbf{R}$ on a pseudo-convex domain D, it is necessary to consider chains of analytic disks joining p and q:

$$(1.2)\qquad d(p, q) = \inf\{\delta(a_1, b_1) + \cdots + \delta(a_n, b_n) : a_i, b_j \in \Delta\}$$