PACIFIC JOURNAL OF MATHEMATICS
Vol. 164, No. 2, 1994

3-VALENT GRAPHS AND THE KAUFFMAN BRACKET

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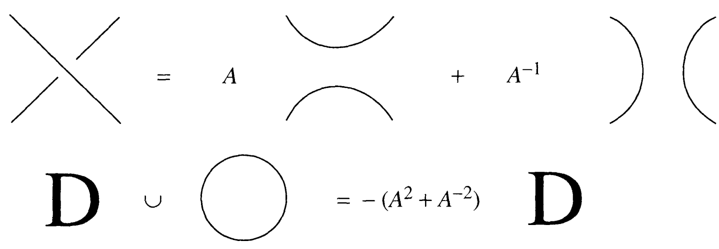
We explicitly determine the tetrahedron coefficient for the one-variable Kauffman bracket, using only Wenzl's recursion formula for the Jones idempotents (or augmentation idempotents) of the Temperley-Lieb algebra.

1. Statement of the main result. In this paper, we consider unoriented knot or tangle diagrams in the plane, up to regular isotopy. Furthermore, we impose the *Kauffman relations* **[Ka1]**



(the first relation refers to three diagrams identical except where shown, and in the second relation, **D** represents any knot or tangle diagram). Coefficients will always be in $\mathbf{Q}(A)$, the field generated by the indeterminate A over the rational numbers. With these relations, (n, n)-tangles (i.e. tangles in the square with n boundary points on the upper edge and n on the lower edge) generate a finite-dimensional associative algebra T_n over $\mathbf{Q}(A)$, called the *Temperley-Lieb algebra* on n strings (see **[Li1]** for more details on what follows). Multiplication in T_n is induced by placing one diagram above another, and the identity element 1_n is given by the identity (n, n)-tangle. Up to isotopy, there is a finite number of (n, n)-tangles without crossings and without closed loops; they form the standard basis of the algebra T_n. Let $\varepsilon: T_n \to \mathbf{Q}(A)$ denote the associated augmentation homomorphism, i.e. $\varepsilon(1_n) = 1$, and ε is zero on the other basis elements. The algebra T_n (with coefficients in $\mathbf{Q}(A)$) is semisimple, and in particular there is an *augmentation idempotent* $f_n \in T_n$ with the property

$$f_n x = x f_n = \varepsilon(x) f_n$$