PACIFIC JOURNAL OF MATHEMATICS
Vol. 168, No. 1, 1995

CONTINUITY OF CONVEX HULL BOUNDARIES

LINDA KEEN AND CAROLINE SERIES

In this paper we consider families of finitely generated Kleinian groups $\{G_\mu\}$ that depend holomorphically on a parameter μ which varies in an arbitrary connected domain in $\mathbb{C}$. The groups G_μ are quasiconformally conjugate. We denote the boundary of the convex hull of the limit set of G_μ by $\partial\mathcal{C}(G_\mu)$. The quotient $\partial\mathcal{C}(G_\mu)/G_\mu$ is a union of pleated surfaces each carrying a hyperbolic structure. We fix our attention on one component S_μ and we address the problem of how it varies with μ. We prove that both the hyperbolic structure and the bending measure of the pleating lamination of S_μ are continuous functions of μ.

1. Introduction. A discrete subgroup $G \subset PSL(2,\mathbb{C})$ is both a subgroup of $\operatorname{aut}(\hat{\mathbb{C}})$ and a group of isometries of hyperbolic 3-space, $\mathbb{H}^3$. The regular set $\Omega = \Omega(G)$ is the subset of $\hat{\mathbb{C}}$ on which the elements of G form a normal family, and the limit set $\Lambda(G)$ is its complement. An important object of study in the Thurston theory of hyperbolic 3-manifolds is the boundary in $\mathbb{H}^3$ of the convex hull of $\Lambda(G)$. This boundary carries all of the essential geometric information about G. Its connected components are examples of what Thurston calls pleated surfaces. They carry an intrinsic hyperbolic metric with respect to which they are complete hyperbolic surfaces. Denote the convex hull boundary by $\partial\mathcal{C} = \partial\mathcal{C}(G)$. Each component of $\partial\mathcal{C}$ "faces" a certain component of Ω; more precisely, each component of $\partial\mathcal{C}$ is the image of a component of Ω under the retraction map defined in Section 2. Topologically, but not conformally, the components of $\partial\mathcal{C}/G$ are equivalent to the components of Ω/G determined by this correspondence.

Suppose now that $\{G_\mu\}$ is a family of Kleinian groups depending holomorphically on a parameter μ that varies in a connected domain $D \subset \mathbb{C}$ in such a way that the groups G_μ are all quasiconformally conjugate. For the sake of readability we often drop the G and