WEIGHTED HADAMARD PRODUCTS OF HOLOMORPHIC FUCTIONS IN THE BALL

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Weighted Hadamard products of holomorphic functions in the unit ball B of $\mathbb{C}^n$ are studied, and are used to establish multiplier theorems for spaces of such functions on B. An interesting feature of such a product of two holomorphic functions f and g on B is that it is holomorphic on the unit polydisk U^n. Moreover, if, in addition, f belongs to the Hardy space $H^1(B)$ and g belongs to the Bloch space $\mathcal{B}(B)$, then the non-weighted Hadamard product of f and g belongs to $\mathrm{BMOA}(U^n)$, the space of holomorphic functions in U^n with bounded mean oscillation on the tours $(\partial U)^n$. Refinements of this result, as well as new charaterizations of spaces of multipliers of holomorphic functions in B, are also established.

1. Introduction. Hadamard products, their properties and related coefficient multipliers problems for spaces of holomorphic functions on the unit disk, are well-known and they have been studied by many authors (see, for example, [5] and the references therein). In the higher dimensional extension of such a study [7, 8] one encounters with several natural, and quite interesting, questions concerning multi-index coefficient multipliers problems and the properties of weighted Haramard products of holomorphic functions of several complex variables. In this paper we shall address these questions in their higher dimensional setting by obtaining new charaterizations, some of which were unexpected, of spaces of multipliers of holomorphic functions in the ball, and in so doing we also extend and refine previously established results.

Before describing these characterizations and their background we need to set up some basic notation which shall also be used throughout the entire paper. By $H(\Omega)$ we denote the space of all holomorphic functions on a domain Ω in $\mathbb{C}^n$. For $z = (z_1, \cdots, z_n) \in$