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JEAN BOURGAIN'S ANALYTIC PARTITION OF UNITY VIA HOLOMORPHIC MARTINGALES

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Using stopping time arguments on holomorphic martingales we present a soft way of constructing J. Bourgain's analytic partition of unity. Further applications to Marcinkiewicz interpolation in weighted Hardy spaces are discussed.

1. Introduction. In his 1984 Acta Mathematica paper Jean Bourgain derives new Banach space properties of H^∞ and the disc algebra from the existence of the following analytic partition of unity:

THEOREM 1 [J. Bourgain]. *Given f, a strictly positive integrable function on $\mathbf{T}$ with $\int f(t)\, dt = 1$ and $0 < \delta < 1$ then, there exist functions $\tau_j, \gamma_j \in H^\infty(T)$ and positive numbers c_j such that:*

1. $\|\gamma_j\|_\infty < C$
2. $\sum |\tau_j| < C$
3. $|\tau_j| f < c_j$
4. $\sum c_j \|\tau_j\|_1 < \delta^{-C}$
5. $\displaystyle\int \left|1 - \sum \gamma_j \tau_j^2\right| f\, dt < \delta.$

Here I wish to present a soft way to this construction which results from using *probabilistic tools* such as holomorphic martingales.

I should like to point out here that a proof for the existence of analytic partitions of unity - much simpler than J. Bourgain's - has been given recently by Serguei Kislyakov. See [**K1**] and [**K2**]. In [**K3**] S. Kislyakov derived J. Bourgain's result on p-summing operators from the following weighted Marcinkiewicz decomposition.

THEOREM 2 [S. Kislyakov]. *For any positive weight b on $\mathbf{T}$ there exists a weight $B \geq b$ and $\int B\, dt < C \int b\, dt$ so that for any $\lambda >$*