PACIFIC JOURNAL OF MATHEMATICS
Vol. 169, No. 1, 1995

VOLUME ESTIMATES FOR LOG-CONCAVE DENSITIES WITH APPLICATION TO ITERATED CONVOLUTIONS

MARIUS JUNGE

A connection between volume estimates for a log-concave, symmetric density of a probability measure on $\mathbb{R}^n$ and its maximal value is established. As an application we prove for an absolute constant c_0

$$\underbrace{f * \cdots * f}_{m \text{ times}}(0) \le \left(\frac{c_0}{\sqrt{m}}\right)^n f(0).$$

0. Introduction. Log-concave densities appear naturally in the theory of convex sets. Besides the normal distributions a lot of information is known about the cube $Q_n = [-1/2, 1/2]^n$. In particular, a modified form of Sudakow's inequality was proved by Carl and Pajor, see [**CP**].

THEOREM 1. *There is an absolute constant c_1, such that for every operator $u : \ell_2^n \to Y$ with* $\mathrm{rg}(u) \le m$ *and all* $k \in \mathrm{N}$

$$\sqrt{k} \max\{d_k(u), e_k(u)\} \le c_1 (\ln(1 + m/k))^{1/2} \int_{Q_n} \|u(x)\|_Y \, dx.$$

Here d_k, e_k denotes the k-th Kolmogorov, entropy numbers, respectively.It is wellknown fact that the logarithmic factor can not be removed.In this paper we are interested in generalizations of Sudakows estimate for an arbitrary log-concave density which is closely related to upper bounds for the maximal value of a log-concave densities.This observation, based on ideas of Hensley and Ball, is contained in the following key

LEMMA 2. *Let $f : \mathbb{R}^n \to \mathbb{R}$ be a log-concave, symmetric density of a probability measure on $\mathbb{R}^n$. There is an absolute constant c_0 such that for all $1 \le p \le 2n$*

$$\frac{1}{c_0} \le f(0)^{1/n} \inf_{B \text{ convex body}} \left(\int_{\mathbb{R}}^n \|x\|_B^p f(x) \, dx \right)^{1/p} \mathrm{vol}(B)^{1/n} \le c_0.$$