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ON THE UNIQUENESS OF CAPILLARY SURFACES OVER AN INFINITE STRIP

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In 1987, Tam proved that the solution of the capillary surface equation without gravity over an infinite strip must be a rigid rotation of a cylinder. Here we give a simple proof for Tam's Theorem and generalize his result.

1. Introduction. Let Ω be a domain in $\mathbb{R}^n$. Consider the equation of prescribed mean curvature

$$\text{div } Tu = H \text{ in } \Omega \tag{1}$$

where $Tu = \frac{Du}{\sqrt{1+|Du|^2}}$ and Du is the gradient of u.

Finn [**4**] proved that if $H = n, \Omega$ contains the unit ball B_1 of $\mathbb{R}^n$, and (1) has a solution u, then Ω has to be exactly B_1 and u must be a lower hemisphere. We emphasize that no boundary condition is imposed.

In the case Ω is unbounded, Finn [**4**] conjectured that the only solution of (1) with $H = 2$ over an infinite strip of width 1 in $\mathbb{R}^2$ is a cylinder. Wang [**9**] and Collin [**2**] independently showed that other different solutions can appear, so Finn's conjecture is not true.

In [**7**], [**8**] Tam considered the problem related to Finn's conjecture as follows:

$$\begin{cases} \text{div } Tu = H & \text{in } \Omega, \\ Tu \cdot \nu = \cos\alpha & \text{on } \partial\Omega \end{cases} \tag{2}$$

where H and α are constants, Ω is the infinite strip $\left(-\frac{1}{2}, \frac{1}{2}\right) \times \mathbb{R}$, ν is the unit outward normal of $\partial\Omega$. The boundary value problem (2) determines the height of a capillary surface without gravity. Note that if (2) has a solution, then $H = 2\cos\alpha$ ([**7**]).