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A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES ON $\mathbf{P}^n$

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This is an addendum to a recent paper of V. Ancona, T. Peternell and J. Wisniewski. Here we prove (using heavily their paper) two criteria for the splitting of rank 2 algebraic vector bundles (one on $\mathbf{P}^n$ and one on certain algebraic complete manifolds).

More precisely, the aim here is to show why the proofs of [**1**, Th. 10.5], and [**1**, Th. 10.13], give the following two theorems.

THEOREM 1. *Let E be a rank 2 algebraic vector bundle on $\mathbf{P}^n$ which satisfies the assumptions of* [**1**, Th. 10.5]. *Then E splits.*

THEOREM 2. *Let E be a rank 2 algebraic vector bundle on a projective manifold X with (X, E) satisfying the assumption of* [**1**, Th. 10.13]. *Then E splits.*

The assumptions on X in Theorem 2 are very restrictive (e.g. X is a Fano manifold with $\mathrm{Pic}(X) \cong \mathbf{Z}$). We only remark that the assumptions of Theorem 1 are satisfied if there is a two dimentional projective family, S, of lines in $\mathbf{P}^n$ such that the splitting type of $E|L$ is the same for all $L \in S$.

Proof of Theorem 1. By the statement of [**1**, Th. 10.13], E numerically splits, i.e. it has the same Chern classes of a direct sum of 2 line bundles, i.e. there are integers a_1, a_2 with $a_1 \leq a_2$ such that $c_1(E) = a_1 + a_2$ and $c_2(E) = a_1 a_2$. The key remark is that the proof of [**1**, Th. 10.5], shows the existence of a line L such that $E|L \cong \mathbf{O}_L(a_1) \oplus \mathbf{O}_L(a_2)$. Since $4c_2(E) - c_1(E)^2 \leq 0$, E is not stable. Hence there is an integer $t \geq (a_1 + a_2)/2$ such that $H^0(\mathbf{P}^n, E(-t)) \neq 0$; take as t the minimal one; the corresponding section s of $E(-t)$ will vanish on a codimension 2 subscheme, Z, with $\deg(Z) = c_2(E(-t))$. Since $c_2(E(-x)) < 0$ if $a_1 < x < a_2$, we have $t \geq a_2$. If $t = a_2$ we obtain $Z = \varnothing$; hence E splits. Hence we