ON A TAUBERIAN THEOREM FOR ABEL SUMMABILITY

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1. Introduction. In 1928 the author proved the following theorem [2, Section 2]:

THEOREM A. *If $p > 1$ and*

$$(1.1) \qquad \sum_{\nu=1}^{n} \nu^p |a_\nu|^p = O(n)\,, \qquad n \longrightarrow \infty\,,$$

then Abel summability of the series $\sum_{n=0}^{\infty} a_n$ to s implies its convergence to s.

The theorem is the more general the smaller p is; it does not hold for $p = 1$ [2, Section 1; 1, pp. 119, 122]. However, for this case Rényi proved the following theorem:

THEOREM B. *If*

$$\lim_{n\to\infty} \frac{1}{n} \sum_{\nu=1}^{n} \nu |a_\nu| = l < \infty$$

exists, then Abel summability of $\sum_{n=0}^{\infty} a_n$ to s implies convergence of the series to s.

2. Generalization. We give a simpler proof and at the same time a slight generalization of Theorem B.

THEOREM 1. *Assume that*

$$(2.1) \qquad V_n = \sum_{\nu=1}^{n} \nu |a_\nu| = O(n)\,,$$

and that

$$(2.2) \qquad \frac{1}{m} V_m - \frac{1}{n} V_n \longrightarrow 0\,,$$

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