THE HEAVY SPHERE SUPPORTED BY A CONCENTRATED FORCE

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1. Introduction. In the linear three-dimensional theory of elasticity only a few particular solutions are known which describe the action of a concentrated force on an isotropic homogeneous solid. The fundamental particular solution which expresses the displacement due to a force at a point within an indefinitely extended solid was given first by Lord Kelvin [5]. It was found again at a later date by Boussinesq [1] along with other particular solutions which can be derived from it and which lead to the solution of the problem of a concentrated force acting on an infinite solid bounded by a plane. Michell [4] obtained the displacements and stresses in an infinite cone acted on by a concentrated force at the vertex by using Boussinesq's results. The solids considered by these authors all extend to infinity.

In this paper a particular solution describing the action of a concentrated force on a finite solid will be considered.

2. The problem. Let there be given an isotropic homogeneous sphere of radius a, which is supported by a radial concentrated force at the south pole. Our problem is the determination of the displacement vector at any point of the sphere in the case of equilibrium, that is, in the case in which the magnitude of the force is equal to the total weight of the sphere.

3. General theory. In the linear theory of elasticity for an isotropic homogeneous medium, the components u, v, w of the displacement vector $\mathfrak{u}$ with respect to a cartesian coordinate system x, y, z satisfy the differential equations of Lamé [2],

$$\Delta \mathfrak{u} + \alpha \operatorname{grad} \operatorname{div} \mathfrak{u} + \beta \mathfrak{X} = 0 \,, \tag{1}$$

where

$$\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \,.$$

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