CONCERNING HEREDITARILY INDECOMPOSABLE CONTINUA

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1. Introduction. A continuum is indecomposable if it is not the sum of two proper subcontinua. It is hereditarily indecomposable if each of its subcontinua is indecomposable. In [3] Knaster gave an example of a hereditarily indecomposable continuum which was not a point. In this paper we study some properties of the Knaster example and describe some other hereditarily indecomposable continua.

2. Chained hereditarily indecomposable continua are homeomorphic. The hereditarily indecomposable continuum given [3] by Knaster was a plane continuum which was described in terms of covering bands. For each positive number ϵ, it could be covered by an ϵ-chain. Moise used [5] a hereditarily indecomposable continuum to exhibit a continuum which was topologically equivalent to each of its nondegenerate subcontinua. He called it a pseudo-arc and noted that it was similar (if not in fact topologically equivalent) to Knaster's example. It could be chained. Bing used [2] such a continuum as an example of a homogeneous plane continuum. Anderson showed [1] that the plane is the sum of a continuous collection of such continua. Theorem 1 reveals that all of these continua are topologically equivalent.

We follow the definitions used in [2]. In particular, we recall the following. A *chain* $D = [d_1, d_2, \cdots, d_n]$ is a collection of open sets $d_1, d_2, \cdots, d_n$ such that d_i intersects d_j if and only if i is equal to $j-1, j$, or $j+1$. If the links are of diameter less than ϵ, the chain is called an ϵ-*chain*. We do not suppose that the links of a chain are necessarily connected.

If the chain $E = [e_1, e_2, \cdots, e_n]$ is a refinement of the chain $D = [d_1, d_2, \cdots, d_m]$, E is called *crooked* in D provided that if $k - h > 2$ and e_i and e_j are links of E in links d_h and d_k of D, respectively, then there are links e_r and e_s of E in links d_{k-1} and d_{h+1}, respectively, such that either $i > r > s > j$ or $i < r < s < j$.

EXAMPLE 1. *The pseudo-arc.* The following description of a chained hereditarily indecomposable continuum appeared in [2] and is much like one given

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