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MULTIPLIERS AND BOURGAIN ALGEBRAS OF $H^\infty + C$ ON THE POLYDISK

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It is well-known that $H^\infty + C$ on the unit circle is a closed subalgebra of $L^\infty(T)$, and Rudin proved the $(H^\infty + C)(T^2)$ is a closed subspace of $L^\infty(T^2)$ but it is not an algebra. The multiplier algebra $\mathcal{M}$ of $(H^\infty + C)(T^2)$ is determined. Using this charaterization, we study Bourgain algebras of type $H^\infty + C$ on the torus T^2 and the polydisk U^2. Both Bourgain algebras of $H^\infty + C$ and $\mathcal{M}$ on the torus coincide with $\mathcal{M}$. We denote by $\tilde{\mathcal{M}}$ the space pf Poisson integral of functions in $\mathcal{M}$ and $C_{T^2}(\bar{U}^2)$ the space of continuous functions on $\bar{U}^2$ which vanish on T^2. It is proved that all higher Bourgain algebras of $H(U^2) + C(\bar{U}^2)$ and $H(U^2) + C_{T^2}(\bar{U}^2)$ are all distinct respectively, but every higher Bourgain algebra of $H(U^2) + C_0(U^2)$ coincides with $H(U^2) + C_0(U^2)$. It is also proved that all higher Bourgain algebras of $\tilde{\mathcal{M}}$ and $\tilde{\mathcal{M}} + C_0(U^2)$ are all distinct respectively, but every higher Bourgain algebra of $\tilde{\mathcal{M}} + C_{T^2}(\bar{U}^2)$ coincides with the first Bourgain algebra of $\tilde{\mathcal{M}} + C_{T^2}(\bar{U}^2)$.

1. Introduction.

Let U^2 be the 2-dimensional unit polydisk and let T^2 be the torus. We denote by $H^\infty(U^2)$ the space of bounded holomorphic functions in U^2 and by $H^\infty(T^2)$ the space of radial limits of functions in $H^\infty(U^2)$. Then $H^\infty(T^2)$ is an essential supremum norm closed subalgebra of $L^\infty(T^2)$, the usual Lebesgue space with respect to $d\theta d\psi/(2\pi)^2$ (see [**12**]). Let denote by $C(X)$ the space of continuous functions on a topological space X. The algebra $A(T^2) = H^\infty(T^2) \cap C(T^2)$ or $A(\bar{U}^2) = H^\infty(U^2) \cap C(\bar{U}^2)$ is called the polydisk algebra, where $\bar{U}^2$ is the closed polydisk. In [**13**, Theorem 2.2], Rudin proved that $(H^\infty + C)(T^2) = H^\infty(T^2) + C(T^2) = \{f + g; f \in H^\infty(T^2), g \in C(T^2)\}$ is a closed subspace of $L^\infty(T^2)$ but it is not an algebra. On the unit circle T, it is well known that $(H^\infty + C)(T)$ is a closed subalgebra of $L^\infty(T)$ [**14**]. Let $\mathcal{M}$ be the space of multipliers of $(H^\infty + C)(T^2)$, that is,

$$\mathcal{M} = \{f \in L^\infty(T^2);\ f \cdot (H^\infty + C)(T^2) \subset (H^\infty + C)(T^2)\}.$$

Then $\mathcal{M}$ is a closed subalgebra of $L^\infty(T^2)$. Since constant functions are contained in $(H^\infty + C)(T^2)$, $\mathcal{M} \subset (H^\infty + C)(T^2)$. Let $C^\infty(U^2)$ be the space of