PACIFIC JOURNAL OF MATHEMATICS
Vol. 171, No. 2, 1995

CORRECTION TO "FREE BANACH-LIE ALGEBRAS, COUNIVERSAL BANACH-LIE GROUPS, AND MORE"

VLADIMIR PESTOV

Vol. **157** (1993), 137–144

We correct a proof of the fact that the free Banach-Lie algebra on a normed space of dimension ≥ 2 is centreless, and observe that, as a corollary, every Banach-Lie algebra is a factor algebra of a Banach-Lie algebra faithfully representable in a Banach space.

1. All the major results of our paper [**2**] are based on the following statement, which appears as a part of Theorem 2.1.

Theorem A. *The free Banach-Lie algebra on a normed space E is either trivial (if $\dim E = 0$), or one-dimensional (if $\dim E = 1$), or centreless.*

Unfortunately, the proof of the above result presented in [**2**] is unsatisfactory, and it was Professor W.T. van Est who has kindly drawn the author's attention to this fact. Below we present a correct proof of Theorem A.

A 1973 investigation [**4**] of van Est and Świerczkowski was partly motivated by the question: is every Banach-Lie algebra a factor algebra of a Banach-Lie algebra faithfully representable in a Banach space? We can answer this in the positive.

Indeed, every Banach-Lie algebra $\mathfrak{g}$ is a factor Banach-Lie algebra of a free Banach-Lie algebra [**2**]. Since centreless Banach-Lie algebras are exactly those whose adjoint representation is faithful, the following direct corollary of Theorem A holds.

Theorem B. *Every Banach-Lie algebra is a factor algebra of a Banach-Lie algebra admitting a faithful representation in a Banach space.*

2. Denote by $\mathbb{K}$ the basic field (either $\mathbb{R}$ or $\mathbb{C}$), and let E be a normed space. For an $n > 0$, let $\mathcal{A}_n(E) = E^{\otimes_\pi n} \equiv E \otimes_\pi E \otimes_\pi \cdots \otimes_\pi E$ be an n-fold (non-completed) projective tensor product. ([**3**, III.6.3.]) Endow the space $\mathcal{B}^n(E)$ of all n-linear continuous functionals on E^n with a norm:

$$\| f \| \stackrel{def}{=} \sup\{|f(x_1, \ldots, x_n)| \colon \| x_i \| \leq 1,\ i = 1, 2, \ldots, n\}.$$