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THE INTRINSIC MOUNTAIN PASS

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We show how the mountain pass and saddle point theorems can be formulated with out the use of "auxiliary" sets. Moreover, we show that results can still be obtained when some basic hypotheses of these theorems are not satisfied. We then apply our results to semilinear problems for partial differential equations.

1. Introduction.

In the mountain pass and saddle point theorems one is concerned with a C^1 functional G on a Banach space E. One wishes to find a solution of $G'(u) = 0$ or at least a sequence $\{u_k\} \subset E$ such that

$$G(u_k) \to c,\ G'(u_k) \to 0 \tag{1.1}$$

for some $c \in R$. A general procedure was formulated in Brezis-Nirenberg [**BN**] as follows. One finds a compact metric space K and selects a closed subset K^* of K such that $K^* \neq \phi, K^* \neq K$. One then picks a map $p^* \in C(K^*, E)$ and defines

$$A = \{p \in C(K, E) : \quad p = p^* \text{ on } K^*\}$$

$$a = \inf_{p \in A} \max_{\xi \in K} G(p(\xi)). \tag{1.2}$$

Brezis-Nirenberg assume
(A) For each $p \in A, \max_{\xi \in K} G(p(\xi))$ is attained at a point in $K \setminus K^*$.
They then prove that there is a sequence satisfying

$$G(u_k) \to a, \quad G'(u_k) \to 0. \tag{1.3}$$

In reference to the procedure one can ask three questions

1. Are the sets K, K^* essential to the method, or can they be eliminated?
2. How can one verify (A)?
3. What can be said if (A) fails to hold?