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REPRODUCING KERNELS AND COMPOSITION SERIES FOR SPACES OF VECTOR-VALUED HOLOMORPHIC FUNCTIONS

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We calculate the norm of each K-type in a vector-valued Hilbert space of holomorphic functions on a tube domain of type I. As a consequence we get composition series of the analytic continuation of certain holomorphic discrete series and an expansion relative to K of the matrix-valued reproducing kernel.

Introduction.

The theory of unitary highest weight modules for semisimple Lie groups is by now very well developed. Questions of classification, intertwining operators and primitive ideals have been settled by algebraic means, see [**1**, **2**, **3**] and [**8**] and reference there. However, there remain some open problems on the analytic side, in particular to find analytical proofs and expressions for the unitarity.

The problem we consider here is to calculate by purely analytical means the invariant Hermitian form in a Harish-Chandra module of highest weight. At the same time, we find explicitly the K-types in a composition series at reducible values of the parameter for the module. This is of interest for example in finding explicit intertwining differential operators giving the various subquotients.

The problem of composition series and expansion of reproducing kernels for analytic continuations of the holomorphic discrete series has been studied extensively. In [**15**] this is done for the type I domain of tube type and for the scalar-valued Bergman spaces by calculating the norm of each K-type. Recall the simplest case, where $G = SU(1,1)$ and the expansion amounts to the binomial formula

$$(1 - z\bar{w})^{-\lambda} = \sum_{k=0}^{\infty} \frac{(\lambda)_k}{k!} z^k \bar{w}^k.$$

Here the monomials z^k, $k \geq 0$, of the complex variable z in the unit disk give the K-types of the corresponding highest weight module, and the coefficients