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# UNIFORM ALGEBRAS GENERATED BY HOLOMORPHIC AND PLURIHARMONIC FUNCTIONS ON STRICTLY PSEUDOCONVEX DOMAINS

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**It is shown that if $f_1, \dots, f_n$ are pluriharmonic functions on a strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$ that are $C^1$ on $\overline{\Omega}$, and the $n \times n$ matrix $(\partial f_j / \partial \overline{z}_k)$ is invertible at every point of $\Omega$, then the norm-closed algebra generated by $A(\overline{\Omega})$ and $f_1, \dots, f_n$ is equal to $C(\overline{\Omega})$.**

## Introduction.

For $K$ a compact set in $\mathbb{C}^n$, let $A(K)$ denote the subalgebra of $C(K)$ consisting of those continuous functions on $K$ that are holomorphic on the interior of $K$. Let $D$ denote the open unit disc in the plane. If $f$ is in $C(\overline{D})$ and $f$ is harmonic but nonholomorphic on $D$, then the norm-closed subalgebra of $C(\overline{D})$ generated by the disc algebra $A(\overline{D})$ and $f$ is equal to $C(\overline{D})$. (See [**I**] for a brief discussion of the history of this result, and [**Č**] and [**A–S**] for two different proofs.) In [**I**] a partial generalization of this result to the ball algebra is obtained: If $f_1, \dots, f_n$ are pluriharmonic on $B_n$ (the open unit ball in $\mathbb{C}^n$) and $C^1$ on $\overline{B}_n$ (i.e., extend to be continuously differentiable on a neighborhood of $\overline{B}_n$), and the $n \times n$ matrix $(\partial f_j / \partial \overline{z}_k)$ is invertible at every point of $B_n$, then the norm-closed subalgebra of $C(\overline{B}_n)$ generated by the ball algebra $A(\overline{B}_n)$ and $f_1, \dots, f_n$ is equal to $C(\overline{B}_n)$. Extensions of this result to more general strictly pseudoconvex domains are presented there as well. It is shown that if the functions $f_1, \dots, f_n$ are assumed to be complex conjugates of holomorphic functions, then the ball can be replaced by an arbitrary strictly pseudoconvex domain. Thus for general functions $f_1, \dots, f_n$ the ball can be replaced by any simply connected strictly pseudoconvex domain. In addition, an argument due to Barnet Weinstock is presented showing that if the functions $f_1, \dots, f_n$ are assumed to be $C^2$, then the ball can be replaced by any strictly pseudoconvex domain with polynomially convex closure. The main purpose of the present paper is to show that the ball can be replaced by an arbitrary strictly pseudoconvex domain without any extra hypotheses on the functions $f_1, \dots, f_n$.

Given complex-valued continuous functions $f_1, \dots, f_k$ on a compact space $X$, we will write $[f_1, \dots, f_k]$ to denote the norm-closed subalgebra of $C(X)$