PACIFIC JOURNAL OF MATHEMATICS
Vol. 172, No. 1, 1996

SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS WITH FULLY NONLINEAR TWO POINT BOUNDARY CONDITIONS II

H.B. THOMPSON

We establish existence results for two point boundary value problems for second order ordinary differential equations of the form $y'' = f(x, y, y')$, $x \in [0,1]$, where f satisfies the Carathéodory measurability conditions and there exist lower and upper solutions. We consider boundary conditions of the form $G((y(0), y(1)); (y'(0), y'(1))) = 0$ for fully nonlinear, continuous G and of the form $(y(i), y'(i)) \in \mathcal{J}(i)$, $i = 0, 1$ for closed connected subsets $\mathcal{J}(i)$ of the plane. We obtain analogues of our results for continuous f. In particular we introduce compatibility conditions between the lower and upper solutions and : (i) G; (ii) the $\mathcal{J}(i)$, $i = 0, 1$. Assuming these compatibility conditions hold and, in addition, f satisfies assumptions guarenteeing a'priori bounds on the derivatives of solutions we show that solutions exist. As an application we generalise some results of Palamides.

1. Introduction.

In this paper we consider two point boundary value problem for second order ordinary differential equations of the form

$$y'' = f(x, y, y'), \text{ for almost all } x \in [0,1] \tag{1.1}$$

where $f : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ satisfies the Carathéodory conditions. By a solution of (1.1) we mean a function y with y' absolutely continuous and y satisfying (1.1) almost everywhere. The first class of boundary conditions we will consider are of the form

$$0 = G((y(0), y(1)); (y'(0), y'(1))), \tag{1.2}$$

where $G : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is continuous. We call boundary conditions of this form fully nonlinear boundary conditions. The second class of boundary conditions we will consider are of the form

$$(y(i), y'(i)) \in \mathcal{J}(i) \text{ for } i = 0, 1, \tag{1.3}$$