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FACTORIZATION OF P-COMPLETELY BOUNDED MULTILINEAR MAPS

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Given Banach spaces $X_1, \dots, X_N$, $Y_1, \dots, Y_N, X, Y$ and subspaces $S_i \subset B(X_i, Y_i)$ $(1 \leq i \leq N)$, we study p-completely bounded multilinear maps $A: S_N \times \cdots \times S_1 \to B(X,Y)$. We obtain a factorization theorem for such A which is entirely similar to the Christensen-Sinclair representation theorem for completely bounded multilinear maps on operator spaces. Our main tool is a generalisation of Ruan's representation theorem for operator spaces in the Banach space setting. As a consequence, we are able to compute the norms of adapted multilinear Schur product maps on $B(\ell_p^n)$.

1. Introduction and preliminaries.

1.1. Introduction. In a recent paper, Pisier [**Pi1**] proved that the Wittstock factorization theorem for completely bounded maps (cf. [**Ha**], [**Pa1**], [**Pa2**], [**W**]) has a natural generalization to the more general framework of p-completely bounded maps defined on sets of Banach space operators. The main goal of this paper is to prove a version of the Christensen-Sinclair theorem (cf. [**CS**, **PS**]) in this extensive setting.

Let us first recall the definition of p-complete boundedness as introduced (or suggested) in [**Pi**]. Let $1 \leq p < +\infty$ be a number. Let X, Y be Banach spaces. We denote by $B(X,Y)$ the space of all bounded operators from X into Y. Let $S \subset B(X,Y)$ be a subspace. We denote by $M_{n,m}(S)$ the vector space of all $n \times m$ matrices with entries from S. Any $s = [s_{ij}] \in M_{n,m}(S)$ may be canonically identified with a bounded operator from $\ell_p^m(X)$ into $\ell_p^n(Y)$. Under this identification, s has the following norm:

$$(1.1) \quad \|[s_{ij}]\|_{M_{n,m}(S)} = \sup \left\{ \left(\sum_i \left\| \sum_j s_{ij}(x_j) \right\|^p \right)^{\frac{1}{p}} / x_1, \dots, x_m \in X, \sum_j \|x_j\|^p \leq 1 \right\}.$$

Then the usual concept of complete boundedness has the following natural extension.