PACIFIC JOURNAL OF MATHEMATICS
Vol. 172, No. 1, 1996

ESTIMATION OF THE NUMBER OF PERIODIC ORBITS

Boju Jiang

The main theme of this paper is to estimate, for self-maps $f : X \to X$ of compact polyhedra, the asymptotic Nielsen number $N^\infty(f)$ which is defined to be the growth rate of the sequence $\{N(f^n)\}$ of the Nielsen numbers of the iterates of f. The asymptotic Nielsen number provides a homotopy invariant lower bound to the topological entropy $h(f)$. To introduce our main tool, the Lefschetz zeta function, we develop the Nielsen theory of periodic orbits. Compared to the existing Nielsen theory of periodic points, it features the mapping torus approach, thus brings deeper geometric insight and simpler algebraic formulation. The important cases of homeomorphisms of surfaces and punctured surfaces are analysed. Examples show that the computation involved is straightforward and feasible. Applications to dynamics, including improvements of several results in the recent literature, demonstrate the usefulness of the asymptotic Nielsen number.

Introduction.

Motivated by dynamical problems, Nielsen theory of fixed points of self-maps $f : X \to X$ of compact polyhedra was generalized to study periodic points, i.e. solutions to $f^n(x) = x$, where f^n is the n-th iterate. See e.g. [**J1**, §III.4]. As the Nielsen number $N(f)$ is a homotopy invariant lower bound to the number of fixed points of f, the Nielsen number $N(f^n)$ is certainly a lower bound to the number of n-points (i.e. fixed points of the n-th iterate) for any map g homotopic to f.

However, generally speaking, the Nielsen numbers are notoriously difficult to compute. We will demonstrate that the asymptotic growth rate of the sequence $\{N(f^n)\}$ (when $n \to \infty$), which we denote by $N^\infty(f)$, is a more computationally accessible invariant than the sequence itself, yet one that is still useful for dynamics. Although the exact evaluation of $N^\infty(f)$ would be desirable, its estimation is a more realistic goal and, as we shall show, one that is sufficient for many applications.

For an asymptotic study, the first challenge is to develop a unified algebraic formulation for the Nielsen theory of all iterates of f so that we can easily relate the fixed point class data of various f^n. This is why we propose the