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M-HYPERBOLIC REAL SUBSETS OF COMPLEX SPACES

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The aim of this paper is to make a first attempt to study real analytic subsets of complex manifolds (or more generally of complex analytic spaces) from the viewpoint of the theory of metric spaces.

1. Introduction.

Our starting point was inspired by the definition of the so-called Kobayashi pseudodistance on complex manifolds We recall briefly that such a pseudodistance is defined on any complex analytic space M using only the space of all holomorphic maps sending the open unit disk Δ in $\mathbb{C}$ in the space M. Moreover the complex space M is said to be "hyperbolic" if such a pseudodistance actually is a real distance, namely it assigns non vanishing values to pair of distinct points of M. In our situation, we introduce a similar pseudodistance $d_{V,M}$ on any subset of V of a complex analytic space M using the space of all holomorphic maps from Δ to M sending the open interval $I =]-1, 1[$ in V, and we introduce the concept of M-hyperbolicity (cf. Section 2).

We are primarily interested in the case when M is a smooth complex manifold and V is a (closed) real analytic smooth submanifold of M, but the definitions work in this more general context as well.

Any holomorphic map between complex manifolds is distance decreasing when the manifolds are endowed with the Kobayashi distances. Our pseudodistances also fulfill this fundamental property. A unexpected phenomenon is that there are some classes of non holomorphic mappings which enjoy this property. A description of such mappings is given in the Section 3 of the paper. As an application, some hyperbolicity criteria are given, and some Liouville type theorems are proved.

We also extend the construction of the Kobayashi-Royden pseudometric when V is a smooth real analytic submanifold of a complex manifold M (Section 4) and we establish some results on the behaviour of a complex Lie group G acting holomorphically on M and leaving V invariant (Section 5). Moreover we define and study the "geodesics" for such a metric Some examples are given (Section 6).