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FACTORIZATION PROBLEMS IN THE INVERTIBLE GROUP OF A HOMOGENEOUS C*-ALGEBRA

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Let X be a compact metric space of dimension d. In previous work, we have shown that for all sufficiently large n, every element of the identity component $U_0(C(X) \otimes M_n)$ of the unitary group $U(C(X) \otimes M_n)$ is a product of at most 4 exponentials of skewadjoint elements. On the other hand, if X is a manifold then some elements of $U_0(C(X) \otimes M_n)$ require at least about d/n^2 exponentials. Similar qualitative behavior (with different bounds: 5 and $d/(2n^2)$) holds for the problem of factoring elements of the identity component $\mathrm{inv}_0(C(X) \otimes M_n)$ of the invertible group as products of exponentials of arbitrary elements of the algebra. In this paper, we identify the sets of finite products of 10 other types of elements of $\mathrm{inv}_0(C(X) \otimes M_n)$, and we show that the minimum lengths of factorizations have the same qualitative behavior as the two exponential factorization problems above (after a suitable minor modification in 3 of the 10 cases). We obtain upper bounds for large n that range from 5 to 22, and lower bounds approximately of the form rd/n^2 with r ranging from 1/16 to 2. The classes of elements we consider all make sense in general unital C*-algebras. They are: unipotents, positive invertibles, selfadjoint invertibles, symmetries, *-symmetries, commutators of elements of $\mathrm{inv}_0(A)$ and $U_0(A)$, accretive elements, accretive unitaries, and positive-stable elements (real part of spectrum positive). The last three classes are the ones requiring the slight modification; without it, lengths of factorization behave like exponential length rather than exponential rank.

Introduction.

If A is a unital C*-algebra, then the C* exponential rank of A, denoted $\mathrm{cer}(A)$, is the smallest $n \in \{1, 1+\varepsilon, 2, 2+\varepsilon, \ldots, \infty\}$ such that every element of the identity componcnt of the unitary group of A is a product of at most n exponentials of skewadjoint elements. (We say u is a product of $k + \varepsilon$ exponentials if it is a limit of products of k exponentials.) In [**13**] it is proved that for $n \geq 2$ and X a compact manifold, $\mathrm{cer}(C(X) \otimes M_n)$ is at least as large as about $\dim(X)/n^2$, but on the other hand is bounded above