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ON NORMS OF TRIGONOMETRIC POLYNOMIALS ON $SU(2)$

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A conjecture about the L^4-norms of trigonometric polynomials on $SU(2)$ is discussed and some partial results are proved.

1. Introduction.

If G is a compact abelian group, an elementary argument shows that $M_p(G) = M_q(G)$ where $M_p(G)$ denotes the space of L^p-multipliers on G and p and q are conjugate indices. Oberlin [7] found a nonabelian totally disconnected compact group G for which $M_p(G) \neq M_q(G)$. Herz [4] conjectured that inequality holds for all those infinite nonabelian compact groups G whose degrees of the irreducible representations are unbounded. However, for compact connected groups, the situation is still unresolved, even for $SU(2)$.

The present paper arose from an attempt to study the Herz conjecture for $SU(2)$. In his unpublished M.Sc. thesis [8], S. Roberts formulated a conjecture for $SU(2)$which, if proved, would settle the Herz conjecture for all compact connected groups. we believe that Robert's conjecture is interesting in its own right as it makes a rather delicate statement connecting the L^p-norms of noncentral trigonometric polynomials with the growth of the Clesh-Gordon coefficients.

We have pursued this interesting conjecture and make some partial progress towards settling it. Our results open the way to a detailed study of some new aspects of L^p analysis on compact Lie groups.

In Section 2, we establish our notation. We state the conjecture in Section 3 and proove some partial results (Theorem 3.2). In Section 4, we show the relevance of the conjecture to Herz's conjecture.

2. Notation and remarks.

2.1. Irreducible representations of $SU(2)$. We summarise some notation and definitions from [6] concerning the irreducible representation of $SU(2)$.