PSEUDO-ANALYTIC VECTORS ON PSEUDO-KÄHLERIAN MANIFOLDS

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1. Introduction. A pseudo-Kählerian manifold is by definition a Riemannian manifold M^{2n} of class C^r $(r\geq 2)$ which has a skew-symmetric tensor field I_{AB}[1] of class C^{r-1} with non-vanishing determinant satisfying following two conditions:

$$(1) \qquad I^A{}_B I^B{}_C = -\delta^A_C\,, \qquad (I^{AB}I_{BC} = -\delta^A_C)$$

$$(2) \qquad I_{AB,C}=0\,,$$

where

$$(3) \qquad I^A{}_B = g^{AE}I_{EB}\,, \qquad I^{AB}=g^{AE}g^{BF}I_{EF}\,,$$

and a comma denotes the covariant differentiation with respect to g_{AB}. It is known that the real representation of a Kählerian manifold of complex dimension n is a pseudo-Kählerian manifold of dimension $2n$ and of class C^ω and the converse is also true. However, the problem whether a pseudo-Kählerian manifold M^{2n} of class C^r $(r\neq\omega)$ can be regarded, by introducing suitable complex coordinate systems on M^{2n}, as a real representation of a (complex) Kählerian manifold or not is, as far as we know, still an open problem. In this paper we shall generalize some theorems which concern analytic vectors on Kählerian manifolds to pseudo-Kählerian manifolds.

2. Definitions of pseudo-analyticity.

DEFINITION 1. A set of *functions* (ϕ, ψ) defined over a pseudo-Kählerian manifold M^{2n} is said to be *pseudo-analytic* if

$$(4) \qquad I^A{}_B\phi_{,A} = \psi_{,B}\,.$$

If (ϕ, ψ) is pseudo-analytic, then $(-\psi, \phi)$ is pseudo-analytic too.

DEFINITION 2. A *contravariant vector field* u^A defined over M^{2n} is said to be *pseudo-analytic* if

$$(5) \qquad I^A{}_B u^B{}_{,C} = u^A{}_{,B} I^B{}_C\,.$$

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[1] We assume that the indices run as follows:

$$\alpha, \beta, \gamma, \cdots = 1, 2, \cdots, n,$$
$$A, B, C, \cdots = 1, 2, \cdots, n, n+1, \cdots, 2n.$$