# ADDITIONAL NOTE ON SOME TAUBERIAN THEOREMS OF O. SZÁSZ

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**1. An additional theorem.** In the note [3] to which this is an addition, Theorem II is exhibited as a generalization of Theorem I and an appeal is made to Szász [6] to indicate the transition from Theorem II to the final result stated as Corollary III′. However, in view of the formal simplicity of Corollary III′ and the wide generality (reflected in its apparent complexity) of Theorem II, it seems worth while to adopt the opposite point of view and record a method, based on the following result, of deducing Theorem II and all related theorems (which cover Szász's) from Corollary III′ [3, p. 384].

THEOREM IV. *If a (real) series $\sum_{n=1}^{\infty} a_n$ is $(\Phi, \lambda)$-summable to $s$, where $\lambda$ denotes the strictly positive increasing divergent sequence $\{\lambda_n\}$ subject to the additional condition $\lambda_{n+1}/\lambda_n \to 1$, and if the series satisfies the Tauberian condition*:

$$(1) \qquad \liminf_{n\to\infty} \frac{1}{\lambda_n} \sum_{\nu=n+1}^{m} \lambda_\nu a_\nu \geq 0\,, \qquad m>n,\ \frac{\lambda_m}{\lambda_n} \to 1\,,$$

*then $\sum_{n=1}^{\infty} a_n$ is convergent to $s$.* (Amnon Jakimovski [1, Theorem 1] gives the case $\phi(u)=e^{-u}$, $\lambda_n=n$.)

*Proof.* We have, by Abel's partial-summation lemma,

$$\sum_{\nu=n+1}^{m} a_\nu = \sum_{\nu=n+1}^{m} \frac{\lambda_\nu a_\nu}{\lambda_\nu} \geq \frac{\lambda_n}{\lambda_{n+1}} \cdot \frac{1}{\lambda_n} \min_{n+1\leq k\leq m} \sum_{\nu=n+1}^{k} \lambda_\nu a_\nu\,.$$

Hence, by (1),

$$\liminf_{n\to\infty} \sum_{\nu=n+1}^{m} a_\nu \geq 0, \qquad m>n, \quad \frac{\lambda_m}{\lambda_n} \to 1.$$

It is well-known [2, p. 33] that the above Schmidt condition is equivalent to the second alternative of hypothesis (12) of Corollary III′ [3, p. 384]. Therefore this corollary establishes that $\sum_{n=1}^{\infty} a_n = s$.

**2. Deductions from Theorem IV.**

COROLLARY IV.1. *In Theorem* IV, (1) *is implied by, and so can be replaced by,* ONE *of the following conditions*:

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