ON GENERATING FUNCTIONS OF THE JACOBI POLYNOMIALS

PETER HENRICI

1. Introduction. The series of Jacobi polynomials

$$\sum_{n=0}^{\infty} a_n \rho^n P_n^{(\nu,\mu)}(\tau) \tag{1}$$

(a_n independent of ρ and τ) has in the case $a_n=1$ already been evaluated by Jacobi in terms of elementary functions, and there are several other known cases where it can be summed explicity. The sum of (1) is then usually called a generating function of the Jacobi polynomials. On the other hand, according to a particular case of a theorem which we have proved recently, every function of a certain class of regular solutions of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{2\mu+1}{x}\frac{\partial u}{\partial x}+\frac{2\nu+1}{y}\frac{\partial u}{\partial y}=0 \tag{2}$$

can be represented by a series of type (1), where

$$\rho=x^2+y^2\,, \qquad \tau=\frac{x^2-y^2}{x^2+y^2}\,, \tag{3}$$

and may therefore be considered as a generating function of the Jacobi polynomials in the above sense. This fact is used in the present paper for the construction of an expansion of type (1) which contains several known results of this kind as special cases. As a side result we shall obtain some identities of Cayley-Orr type between the coefficients in the Taylor expansions of certain products of hypergeometric series.

In what follows x and y are considered as independent complex variables. Also the variables

$$z=x+iy\,, \qquad z^*=x-iy \tag{4}$$

will be used. Our notation of special functions is in accordance with [5].

2. The expansion theorem. The special case $k=0$ of the main theorem of [6] is as follows:

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