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TWO-GROUPS AND JORDAN ALGEBRAS

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Stroud and Paige have introduced an important class of central simple Jordan algebras $B(2^n)$ of characteristic two. This paper determines the automorphism groups of the algebras $B(2^n)$ and, in so doing, produces an infinite family of finite 2-groups. This is accomplished by characterizing the automorphisms of $B(2^n)$ as matrices operating on the natural basis for the underlying vector space of $B(2^n)$ and then using this characterization to obtain generators and commuting relations for the automorphism groups.

Throughout the paper let $q = 2^{n-2}$, $r = 2^{n-1}$, $s = 2^n$, and $t = 2^{n+1}$. $\delta_{i,j}$ is the Kronecker delta.

1. The algebras.

In 1965 J. B. Stroud [3], pursuing some earlier work of E. C. Paige [2], defined the following class of vector spaces and proved that they are central simple Jordan Algebras of characteristic two:

DEFINITION. Let $B(2^n)$ for $n \geqq 2$ be the vector space over the field Z_2 of two elements with basis $u_{-1}, u_0, u_1, \cdots, u_{s-2}, v_1, v_2, \cdots, v_s$ and with multiplication in $B(2^n)$ defined inductively as follows:

The products u_iu_j for $-1 \leqq i, j \leqq s-2$ are defined by:

(1) $u_0u_i = u_i$ for $-1 \leqq i \leqq s-2$,

(2) $u_{-1}^2 = 0$, $u_{-1}u_i = u_{i-1}$ for $0 \leqq i \leqq s-2$,

(3) $u_iu_j = u_ju_i$ for $-1 \leqq i, j \leqq s-2$,

(4) $u_1^2 = u_2^2 = u_1u_2 = 0$.

Assuming the products u_iu_j are defined for $-1 \leqq i, j \leqq 2^k - 2$ where $2 \leqq k \leqq n-1$, let $p = 2^k$ and define

(5) $u_{p+i}u_j = H_{i,j}u_{p+i+j}$ when $u_iu_j = H_{i,j}u_{i+j}$, $j \neq -1$ with $H_{i,j}$ in Z_2,

(6) $u_{p+i}u_{p+j} = 0$.

For $k \geqq 1$, $m \geqq 0$, $p = 2^k$, define the products u_iv_j by:

(7) $u_iv_j = v_ju_i$ for $-1 \leqq i \leqq s-2$, $1 \leqq j \leqq s$,

(8) $u_0v_j = v_j$ for $1 \leqq j \leqq s$,

(9) $u_{-1}v_j = v_j + v_{j-1}$, $v_0 = 0$ for $1 \leqq j \leqq s$,

(10) $u_{p-1}v_{(2m)p+j} = v_{(2m+1)p+j-1} + v_{(2m+1)p+j}$ for $1 \leqq j \leqq p$,

(11) $u_{p+i}v_{(2m)p+j} = d_{i,j}v_{(2m+1)p+i+j} + e_{i,j}v_{(2m+1)p+i+j+1}$

when

(12) $u_iv_j = d_{i,j}v_{i+j} + e_{i,j}v_{i+j+1}$ for $0 \leqq i \leqq p-2$, $1 \leqq j \leqq p$ and $d_{i,j}$, $e_{i,j}$ in Z_2,

(13) $u_{p+i}v_{(2m+1)p+j} = 0$ for $-1 \leqq i \leqq p-2$, $1 \leqq j \leqq p$.