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DYNAMICAL SYSTEMS OF CHARACTERISTIC 0^+

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The main purpose of this paper is to classify the dynamical systems on the plane which satisfy a certain type of stability criterion. Such flows are referred to as dynamical systems of characteristic 0^+. The classification is based on the consideration of three mutually exclusive and exhaustive cases: Dynamical systems of characteristic 0^+ which have no critical points. Those whose critical points form nonempty compact sets, and those whose critical points do not form compact sets.

Dynamical systems of characteristic 0^+ are those dynamical systems in which all closed positively invariant sets are positively D-stable, i.e., stable in Ura's sense (see [11]). If the phase space of a flow is regular, then a closed positively invariant set, which is positively stable in Liapunov's sense, is also positively D-stable. Thus, some simple examples of flows of characteristic 0^+ are those where the phase spaces are regular and all closed invariant sets are positively stable in Liapunov's sense.

In § 2 we give some of the basic definitions and notations that are used throughout the paper. In § 3 we prove some results of a more general nature which are later applied to flows of characteristic 0^+ on the plane. It is proved that if the phase space X of a flow is normal and connected and a closed invariant set F is globally $+$ asymptotically stable, then F is connected. Further, if the phase space X of a flow of characteristic 0^+ is connected and locally compact, then a compact subset M of X is a positive attractor implies that M is globally $+$ asymptotically stable.

In § 4 we discuss flows of characteristic 0^+ on the plane. It is shown that if the set of critical points S of such a flow is empty, then the flow is parallelizable. If S is compact, then it either consists of a single point which is a Poincaré center, or it is globally $+$ asymptotically stable. If S is not compact, then either $R^2 = S$, or S is $+$ asymptotically stable; S and the region of positive attraction $A^+(S)$ of S each has a countable number of components. Further, each component of $A^+(S)$ is homeomorphic to R^2. At the end of this section, we summarize all the results of this section in the form of a complete classification of such flows.

In § 5 we discuss flows of characteristic $0^\pm$ on the plane, i.e., those in which every closed invariant set is positively and negatively stable in Ura's sense. We prove that such a flow is either parallelizable, or it has a single critical point which is a global Poincaré center, or all