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STRONGLY REGULAR MAPPINGS WITH COMPACT *ANR* FIBERS ARE HUREWICZ FIBERINGS

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We prove that a strongly regular map $f: E \to B$ with compact separable *ANR* fibers and complete separable finite dimensional base space B is a Hurewicz fibering. We also show that if f is a Hurewicz fiber map with locally compact *ANR* fibers and base, then the total space E is an *ANR*.

1. Statement of results. In this paper, we will be concerned with the following two questions:

Q1: When is a map between metric spaces a Hurewicz fibration?

Q2: ([2]) Is the total space of a Hurewicz fibration an *ANR* if the base space is an *ANR* and the fibers are *ANR* s?

Both of these questions have been answered in the finite dimensional case, but our interest in Hilbert cube manifolds has led us to consider the case of locally compact *ANR* s. Finite dimensional answers to Q1 may be found in [1] and [16], while the finite dimensional answer to Q2 may be found in [2]. Weaker infinite dimensional results may be found in [14] and [15].

The notion of a strongly regular map, introduced in [1], is a key ngredient in our work on Q1. Intuitively, a map is strongly regular if nearby point-inverses are homotopy equivalent via small homotopy equivalences. Here is a formal definition.

DEFINITION. A map $f: X \to B$ between metric spaces is said to be *strongly regular* if f is proper ($f^{-1}(K)$ is compact for each compact $K \subset B$) and if for each $b \in B$ and $\epsilon > 0$ there is a $\delta > 0$ such that if $d(b, b') < \delta$ then there are maps $g_{bb'}: f^{-1}(b) \to f^{-1}(b')$ and $g_{b'b}: f^{-1}(b') \to f^{-1}(b)$ and homotopies $h_t: f^{-1}(b) \to f^{-1}(b)$ and $k_t: f^{-1}(b') \to f^{-1}(b')$ such that

(i) $d(g_{bb'}(x), x) < \epsilon$ and $d(h_t(x), x) < \epsilon$ for all $x \in f^{-1}(b)$ and for all t, $0 \leqq t \leqq 1$.

(ii) $d(g_{b'b}(x), x) < \epsilon$ and $d(k_t(x), x) < \epsilon$ for all $x \in f^{-1}(b')$ and for all t, $0 \leqq t \leqq 1$.

(iii) $h_0 = g_{b'b} \circ g_{bb'}$ and $h_1 = id$.

(iv) $k_0 = g_{bb'} \circ g_{b'b}$ and $k_1 = id$.

Here are our main results.

THEOREM 1. *If $f: E \to B$ is a strongly regular map onto a complete*