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SOME THEOREMS ON GENERALIZED DEDEKIND–RADEMACHER SUMS

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Radamacher has defined a generalized Dedekind sum

$$s(h,k;x,y)=\sum_{a \pmod k}\left(\left(h\frac{a+y}{k}+x\right)\right)\left(\left(\frac{a+y}{k}\right)\right)$$

and proved a reciprocity theorem for this sum that generalizes the well known result for $s(h,k)$. In the present paper we define

$$\phi_{r,s}(h,k;x,y)=\sum_{a \pmod k}\bar{B}_r\left(h\left(\frac{a+y}{k}\right)+x\right)\bar{B}_s\left(\frac{a+y}{k}\right),$$

$$\psi_{r,s}(h,k;x,y)=\sum_{j=0}^{r}(-1)^{r-j}\binom{r}{j}h^{r-j}\phi_{j,r+s-j}(h,k;x,y),$$

where $\bar{B}_n(x)$ is the Bernoulli function, and show that

$$\begin{aligned}&(s+1)k^s\psi_{r+1,s}(h,k;x,y)-(r+1)h^r\psi_{s+1,r}(k,h;y,x)\\&\quad=(s+1)k\bar{B}_{r+1}(x)\bar{B}_s(y)-(r+1)h\bar{B}_r(x)\bar{B}_{s+1}(y)\qquad((h,k)=1).\end{aligned}$$

We also prove the polynomial reciprocity theorem

$$(1-v)\sum_{a=0}^{k-1}u^{h-[(ha+z)/k]}v^a-(1-u)\sum_{b=0}^{h-1}v^{k-[(kb+z)/h]}u^b=u^h-v^k \qquad ((h,k)=1)$$

as well as some related results.

1. Introduction.

For real x put

$$((x))=\begin{cases}x-[x]-\frac{1}{2} & (x\neq \text{integer})\\ 0 & (x=\text{integer}),\end{cases}$$

where $[x]$ denotes the greatest integer $\leqq x$. The Dedekind sum $s(h,k)$ is defined by