CHAPTER V

27. Statement of the Result Proved in Chapter V

The following result is proved in this chapter.

THEOREM 27.1. *Let $\mathfrak{G}$ be a minimal simple group of odd order. Then $\mathfrak{G}$ satisfies the following conditions*:

(i) *p and q are odd primes with $p > q$. $\mathfrak{G}$ contains elementary abelian subgroups $\mathfrak{P}$ and $\mathfrak{Q}$ with $|\mathfrak{P}| = p^q$, $|\mathfrak{Q}| = q^p$. $\mathfrak{P}$ and $\mathfrak{Q}$ are* T.I. *sets in $\mathfrak{G}$.*

(ii) *$N(\mathfrak{P}) = \mathfrak{P}\mathfrak{U}\mathfrak{Q}^*$, where $\mathfrak{P}\mathfrak{U}$ and $\mathfrak{U}\mathfrak{Q}^*$ are Frobenius groups with Frobenius kernels $\mathfrak{P}, \mathfrak{U}$ respectively. $|\mathfrak{Q}^*| = q$, $|\mathfrak{U}| = (p^q - 1)/(p - 1)$, $\mathfrak{Q}^* \subseteq \mathfrak{Q}$ and $((p^q - 1)/(p - 1), p - 1) = 1$.*

(iii) *If $\mathfrak{P}^* = C_{\mathfrak{P}}(\mathfrak{Q}^*)$, then $|\mathfrak{P}^*| = p$ and $\mathfrak{P}^*\mathfrak{Q}^*$ is a self-normalizing cyclic subgroup of $\mathfrak{G}$. Furthermore, $C(\mathfrak{P}^*) = \mathfrak{P}\mathfrak{Q}^*$, $C(\mathfrak{Q}^*) = \mathfrak{Q}\mathfrak{P}^*$, and $\mathfrak{P}^* \subseteq N(\mathfrak{Q})$.*

(iv) *$C(\mathfrak{U})$ is a cyclic group which is a* T.I. *set in $\mathfrak{G}$. Furthermore, $\mathfrak{Q}^* \subseteq N(\mathfrak{U}) = N(C(\mathfrak{U}))$, $N(\mathfrak{U})/C(\mathfrak{U})$ is a cyclic group of order pq and $N(\mathfrak{U})$ is a Frobenius group with Frobenius kernel $C(\mathfrak{U})$.*

In this chapter we take the results stated in Section 14 as our starting point. The notation introduced in that section is also used. There is no reference to any result in Chapter IV which is not contained in Section 14. The theory of group characters plays an essential role in the proof of Theorem 27.1. In particular we use the material contained in Chapter III.

Sections 28–31 consist of technical results concerning the characters of various subgroups of $\mathfrak{G}$. In Section 32 the troublesome groups of type V are eliminated. In Section 33 it is shown that groups of type I are Frobenius groups. By making use of the main theorem of [10] it is then easy to show that the first possibility in Theorem 14.1 cannot occur. The rest of the chapter consists of a detailed study of the groups $\mathfrak{S}$ and $\mathfrak{T}$ until in Section 36 we are able to supply a proof of Theorem 27.1.

28. Characters of Subgroups of Type I

Hypothesis 28.1.

(i) *$\mathfrak{X}$ is of Frobenius type with Frobenius kernel $\mathfrak{H}$ and complement $\mathfrak{E}$.*

(ii) *$\mathfrak{E} = \mathfrak{A}\mathfrak{B}$, where $\mathfrak{A}$ is abelian, $\mathfrak{B}$ is cyclic, and $(|\mathfrak{A}|, |\mathfrak{B}|) = 1$.*