CHAPTER III

9. Tamely Imbedded Subsets of a Group

The character ring of a group has a metric structure which is derived from the inner product. Let $\mathfrak{L}$ be a subgroup of the group $\mathfrak{X}$. The purpose of this chapter is to state conditions on $\mathfrak{L}$ and $\mathfrak{X}$ which ensure the existence of an isometry τ that maps suitable subsets of the character ring of $\mathfrak{L}$ into the character ring of $\mathfrak{X}$ and has certain additional properties. If α is in the character ring of $\mathfrak{L}$ and α^τ is defined then these additional properties will yield information concerning $\alpha^\tau(L)$ for some elements L of $\mathfrak{L}$. Once the existence of τ is established it will enable us to derive information about certain generalized characters of $\mathfrak{X}$ provided we know something about the character ring of $\mathfrak{L}$. In this way it is possible to get global information about $\mathfrak{X}$ from local information about $\mathfrak{L}$.

There are two stages in establishing the existence of τ. First we will require that $\mathfrak{L}$ is in some sense "nicely" imbedded in $\mathfrak{X}$. When this requirement is fulfilled it is possible to define α^τ for certain generalized characters α of $\mathfrak{L}$ with $\alpha(1) = 0$. In this situation α^τ is explicitly defined in terms of induced characters of various subgroups of $\mathfrak{X}$. Secondly it is necessary that the character ring of $\mathfrak{L}$ have certain special properties. These properties make it possible to extend the definition of τ to a wider domain. In particular it is then possible to define α^τ for some generalized characters α of $\mathfrak{L}$ with $\alpha(1) \neq 0$. The precise conditions that the character ring of $\mathfrak{L}$ needs to satisfy will be stated later. In this section we are concerned with the imbedding of $\mathfrak{L}$ in $\mathfrak{X}$. The following definition is appropriate.

DEFINITION 9.1. Let $\hat{\mathfrak{L}}$ be a subset of the group $\mathfrak{X}$ such that

$$\langle 1 \rangle \subseteq \hat{\mathfrak{L}} \subseteq N(\hat{\mathfrak{L}}) = \mathfrak{L} \, . \tag{9.1}$$

Let $\mathfrak{L}_0$ be the set of elements L in $\hat{\mathfrak{L}}$ such that $C(L) \subseteq \mathfrak{L}$, and let $\mathfrak{D} = \hat{\mathfrak{L}}^\# - \mathfrak{L}_0$.

We say that $\hat{\mathfrak{L}}$ is *tamely imbedded* in $\mathfrak{X}$ if the following conditions are satisfied:

(i) If two elements of $\hat{\mathfrak{L}}$ are conjugate in $\mathfrak{X}$, they are conjugate in $\mathfrak{L}$.

(ii) If $\mathfrak{D}$ is non empty, then there are non identity subgroups $\mathfrak{H}_1, \cdots, \mathfrak{H}_n$ of $\mathfrak{X}$, $n \geq 1$, with the following properties: