

SOLVABILITY OF GROUPS OF ODD ORDER

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CHAPTER I

1. Introduction

The purpose of this paper is to prove the following result:

THEOREM. *All finite groups of odd order are solvable.*

Some consequences of this theorem and a discussion of the proof may be found in [11].

The paper contains six chapters, the first three being of a general nature. The first section in each of Chapters IV and V summarizes the results proved in that chapter. These results provide the starting point of the succeeding chapter. Other than this, there is no cross reference between Chapters IV, V and VI. The methods used in Chapter IV are purely group theoretical. The work in Chapter V relies heavily on the theory of group characters. Chapter VI consists primarily of a study of generators and relations of a special sort.

2. Notation and Definitions

Most of the following lengthy notation is familiar. Some comes from a less familiar set of notes of P. Hall [20], while some has arisen from the present paper. In general, groups and subsets of groups are denoted by German capitals, while group elements are denoted by ordinary capitals. Other sets of various kinds are denoted by English script capitals. All groups considered in this paper are finite, except when explicitly stated otherwise.

Ordinary lower case letters denote numbers or sometimes elements of sets other than subsets of the group under consideration. Greek letters usually denote complex valued functions on groups. However,

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